

Chapter 5
Operational amplifiers.
Applications

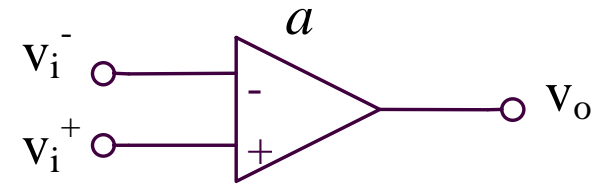
5.1. Introduction

5.1. Introduction

It is an amplifier with:

- high voltage gain (ideal ∞)
- large input impedance (ideal ∞)
- small output impedance (ideal 0)
- differential inputs and single output

$$v_O = a(v_i^+ - v_i^-)$$



Parameters

- voltage gain
- input offset voltage (DC voltage applied between the inputs for obtaining a null output voltage)
- the input offset current

Order of magnitude

$$a_{v0} > 10^5$$

$$V_{IO} = 2-10 \text{ mV}$$

$$I_{IO} = 5\text{nA} - \text{bipol.}$$

$$I_{IO} = 1\text{pA} - \text{CMOS}$$

Parameters

- input biasing currents
- input impedance
- common-mode input range
- output voltage range
- common-mode rejection ratio (differential voltage gain/
common-mode voltage gain)
- output impedance
- bandwidth (frequency for which the voltage gain without
reaction is equal with 1)
- slew-rate (maximum slope of a transient response for a
large signal input “rectangular” signal)

Order of magnitude

$$I_I = 80 \text{ nA} - \text{bipol.}$$

$$I_I = 10 \text{ pA} - \text{CMOS}$$

$$R_i = 2\text{M}\Omega$$

$$R_i \rightarrow \infty - \text{CMOS}$$

$$V_{CC} (V_{DD}) - 2\text{V}$$

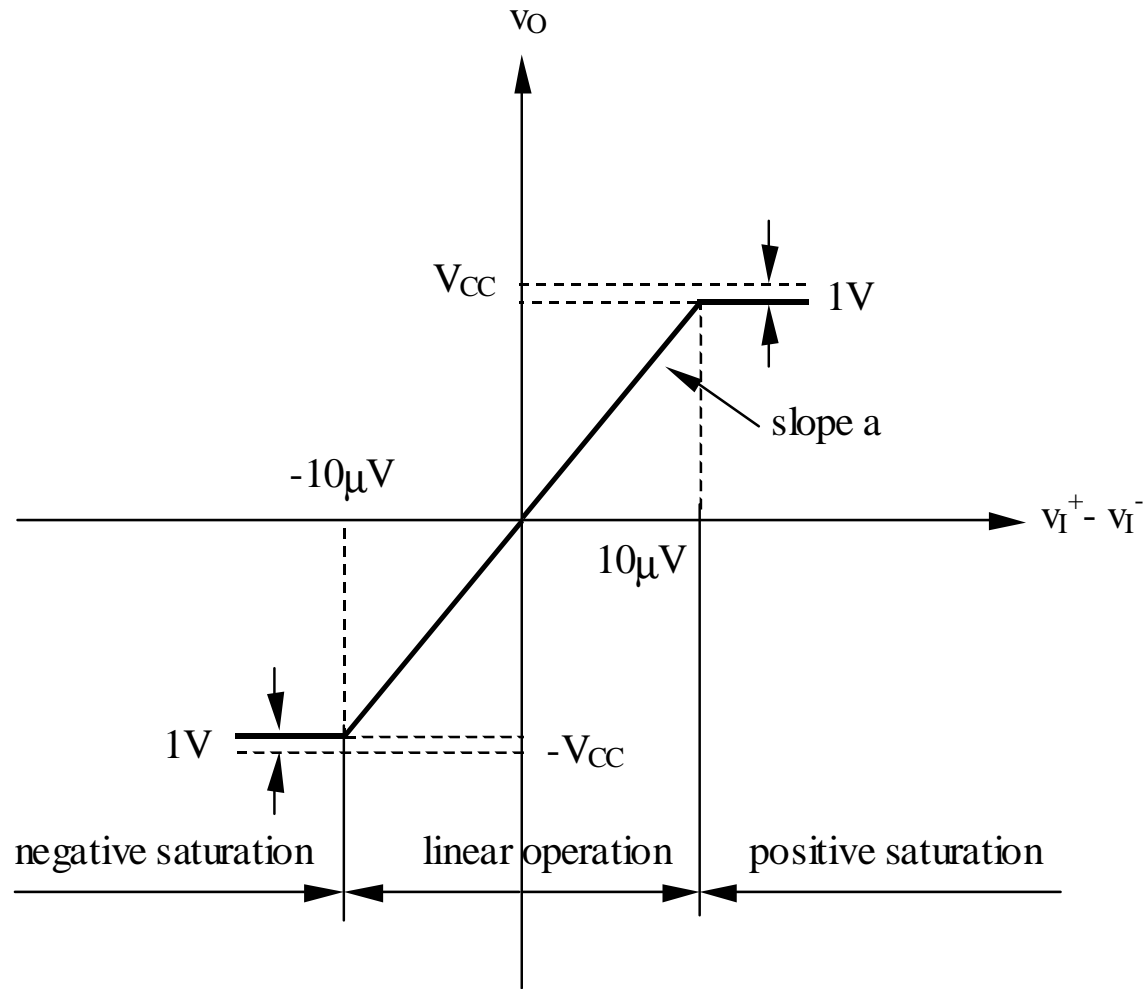
$$V_{CC} (V_{DD}) - 1\text{V}$$

$$\text{CMRR} = 80\text{dB}$$

$$R_O = 50\Omega$$

$$f_T = 1\text{MHz}$$

$$\text{SR} = 0.5\text{V}/\mu\text{s}$$



In linear region, $v_O = a(v_I^+ - v_I^-)$

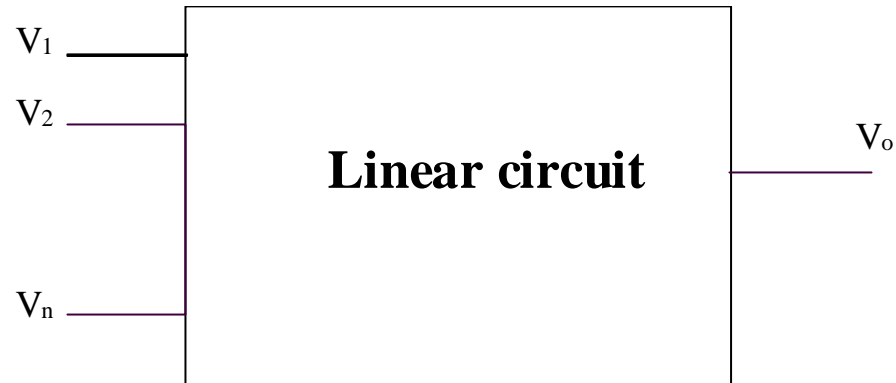
In negative saturation, $v_O \cong -(V_{CC} - IV)$

In positive saturation, $v_O \cong V_{CC} - IV$

5.2. Applications with operational amplifiers

5.2. Applications with operational amplifiers

Superposition theorem



$$V_o = V_o \Big|_{\substack{V_1 \neq 0 \\ V_2 = V_3 = \dots = V_n = 0}} + V_o \Big|_{\substack{V_2 \neq 0 \\ V_1 = V_3 = \dots = V_n = 0}} + \dots + V_o \Big|_{\substack{V_n \neq 0 \\ V_1 = V_2 = \dots = V_{n-1} = 0}}$$

5.2. Applications with operational amplifiers

5.2.1. Ideal operational amplifier

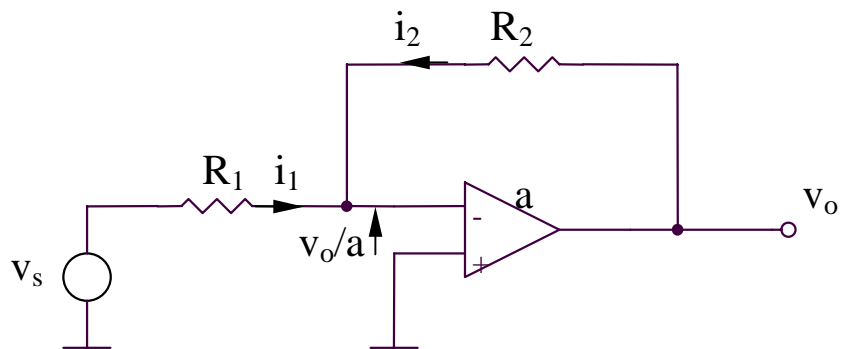
An ideal operational amplifier is characterized in open loop by:

- infinite voltage gain
- infinite input impedance
- null output impedance

In consequence:

- the differential voltage between the two input pins is zero
- the input currents are zero

5.2.2. Inverting amplifier



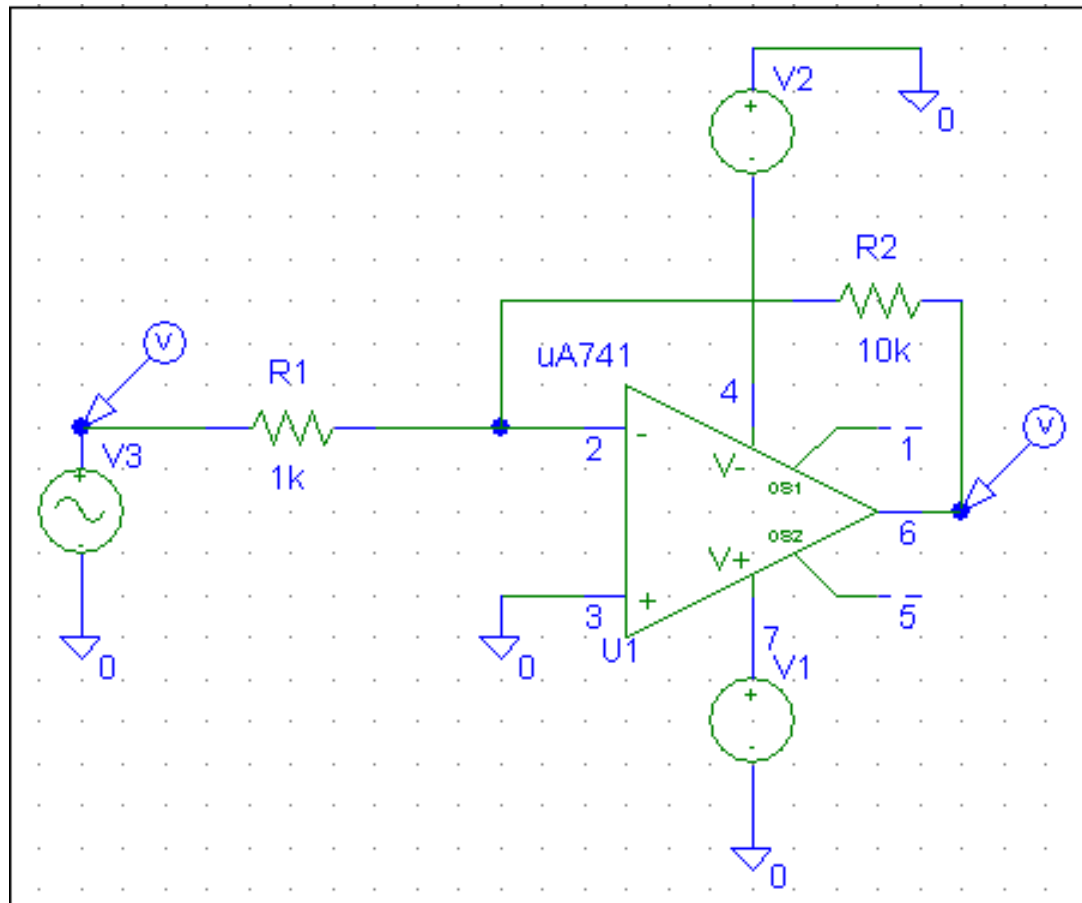
$$\frac{v_s + \frac{v_o}{a}}{R_1} + \frac{v_o + \frac{v_o}{a}}{R_2} = 0 \Rightarrow$$

$$\Rightarrow A = \frac{v_o}{v_s} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{1}{a} \frac{R_1 + R_2}{R_1}} \rightarrow -\frac{R_2}{R_1}$$

SIMULATIONS for inverting amplifier

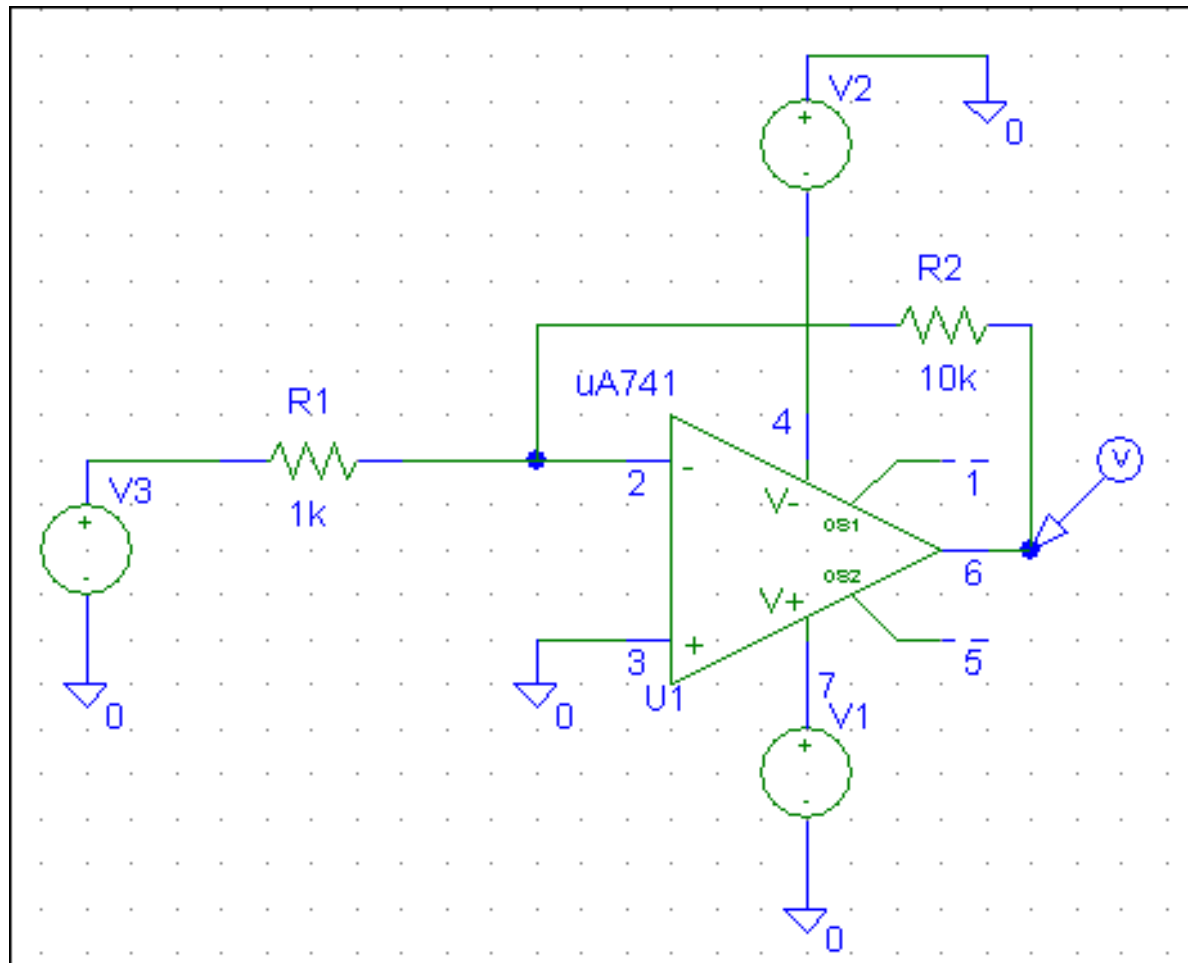
SIMULATIONS for inverting amplifier

SIM 5.1: $V_3(t)$, $V_O(t)$

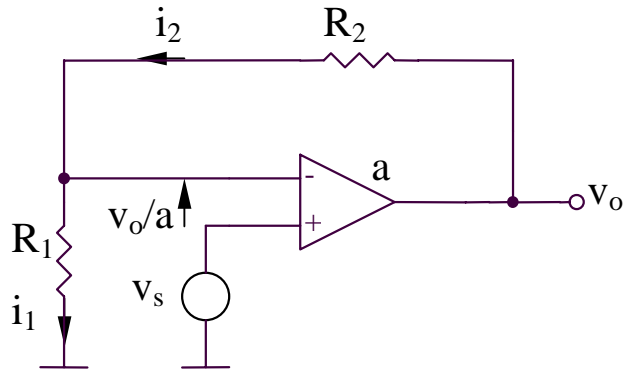


SIMULATIONS for inverting amplifier

SIM 5.2: V_O (V_3)



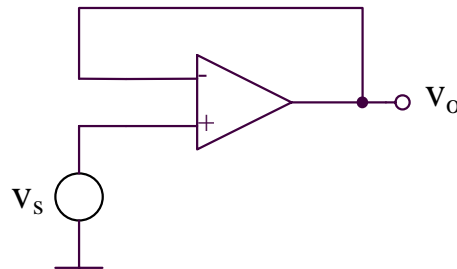
5.2.3. Non-inverting amplifier



$$\frac{v_s - \frac{v_o}{a}}{R_1} = \frac{v_o - \left(v_s - \frac{v_o}{a}\right)}{R_2} \Rightarrow$$

$$\Rightarrow A = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1} \frac{1}{1 + \frac{R_1 + R_2}{aR_1}} \rightarrow 1 + \frac{R_2}{R_1}$$

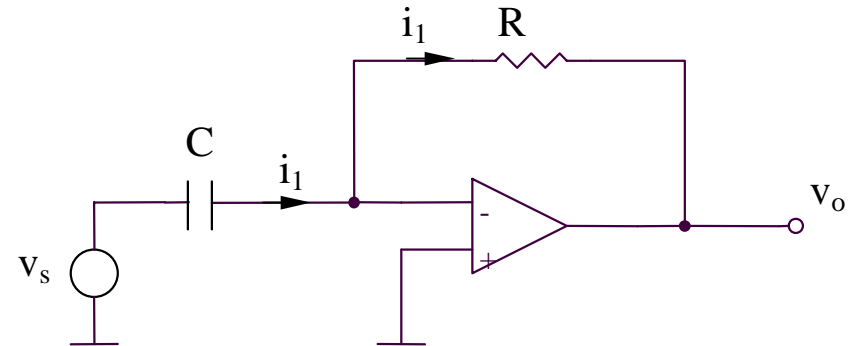
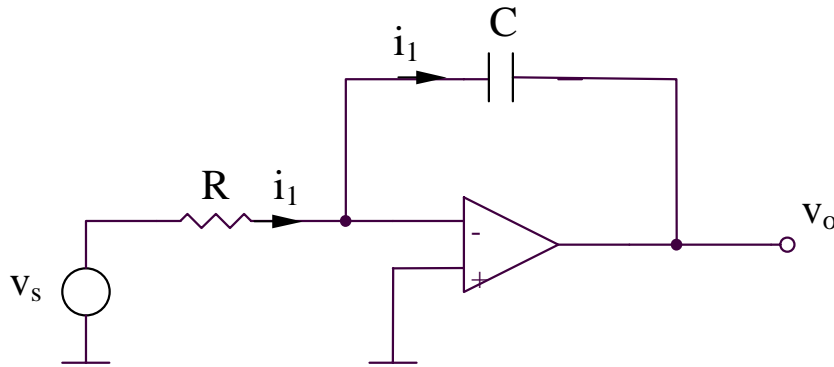
5.2.4. Follower circuit



If the operational amplifier is ideal, the output voltage is identical with the input voltage:

$$v_o = v_s$$

5.2.5. Integrating and differentiating circuits



Integrating circuit

$$i_1 = \frac{v_s(t)}{R}$$

$$v_o = -\frac{1}{C} \int i_1(t) dt + v_o(0)$$

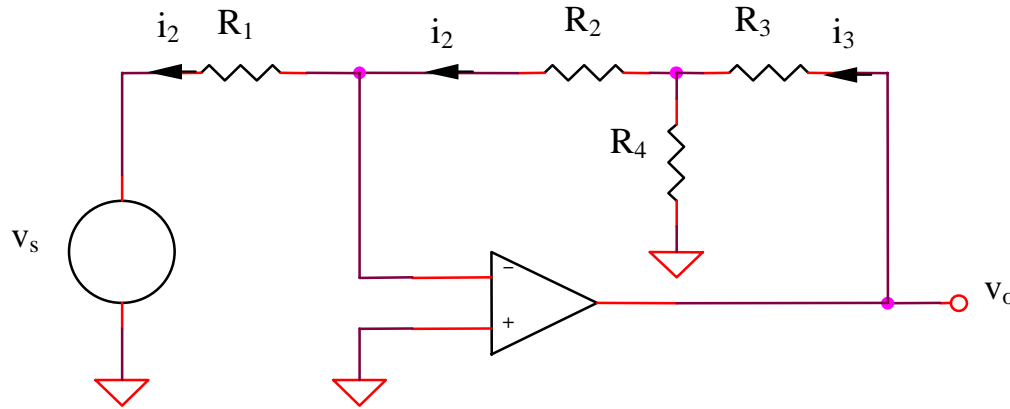
$$v_o = -\frac{1}{RC} \int v_s(t) dt + v_o(0)$$

Differentiating circuit

$$i_1 = C \frac{dv_s}{dt}$$

$$v_o = -Ri_1 = -RC \frac{dv_s}{dt}$$

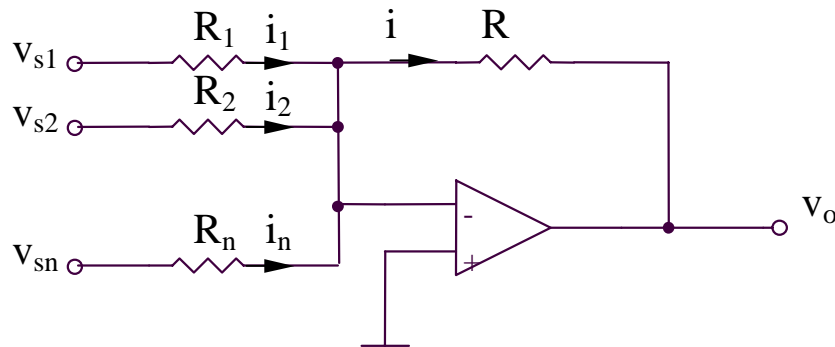
5.2.6. High gain amplifier



$$A = \frac{v_o}{v_s} = \frac{v_o}{i_3} \frac{i_3}{i_2} \frac{i_2}{v_s}$$

$$A = -\frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_1 R_4}$$

5.2.7. Inverting voltage adder



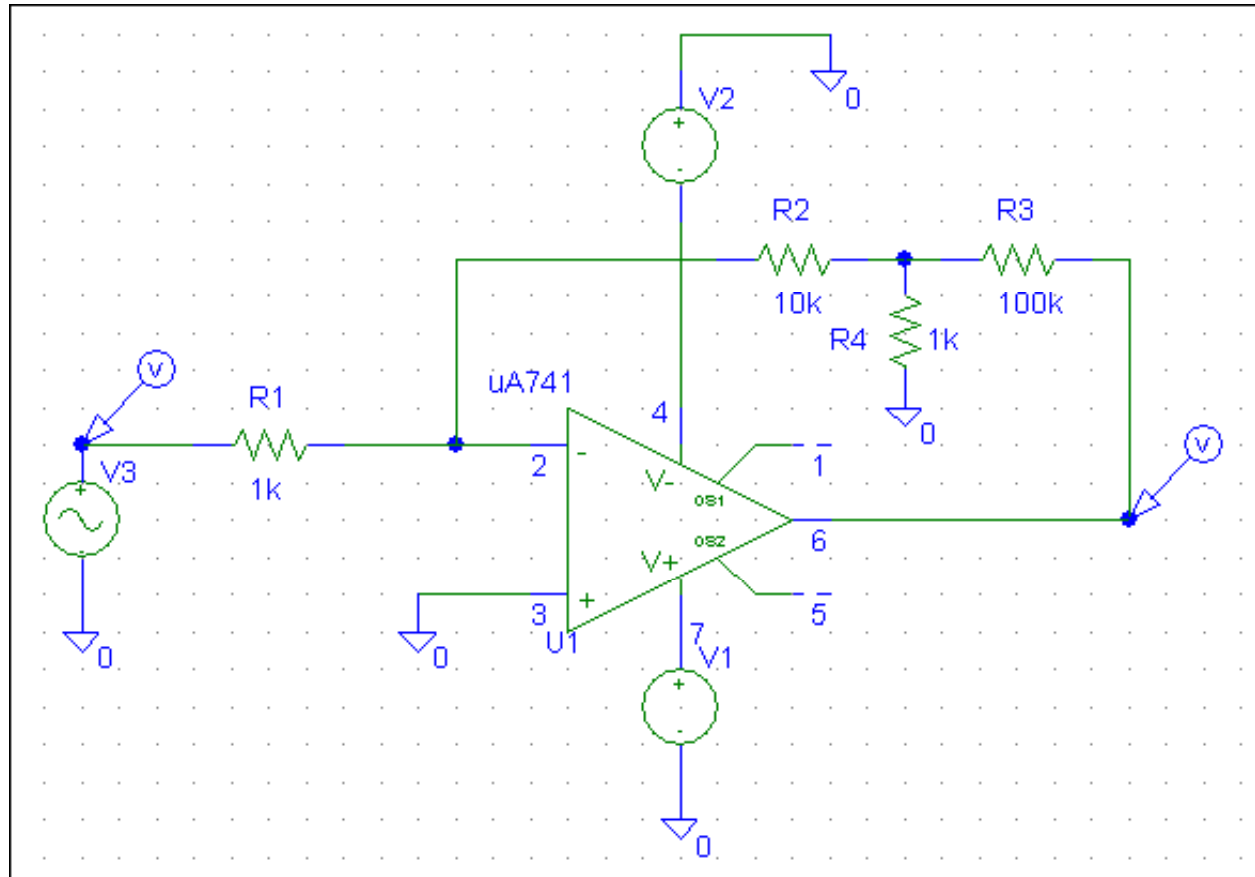
$$i = \sum_{i=1}^n i_i = \sum_{i=1}^n \frac{v_{si}}{R_i}$$

$$v_o = -Ri = -R \sum_{i=1}^n \frac{v_{si}}{R_i}$$

SIMULATIONS for high gain amplifier

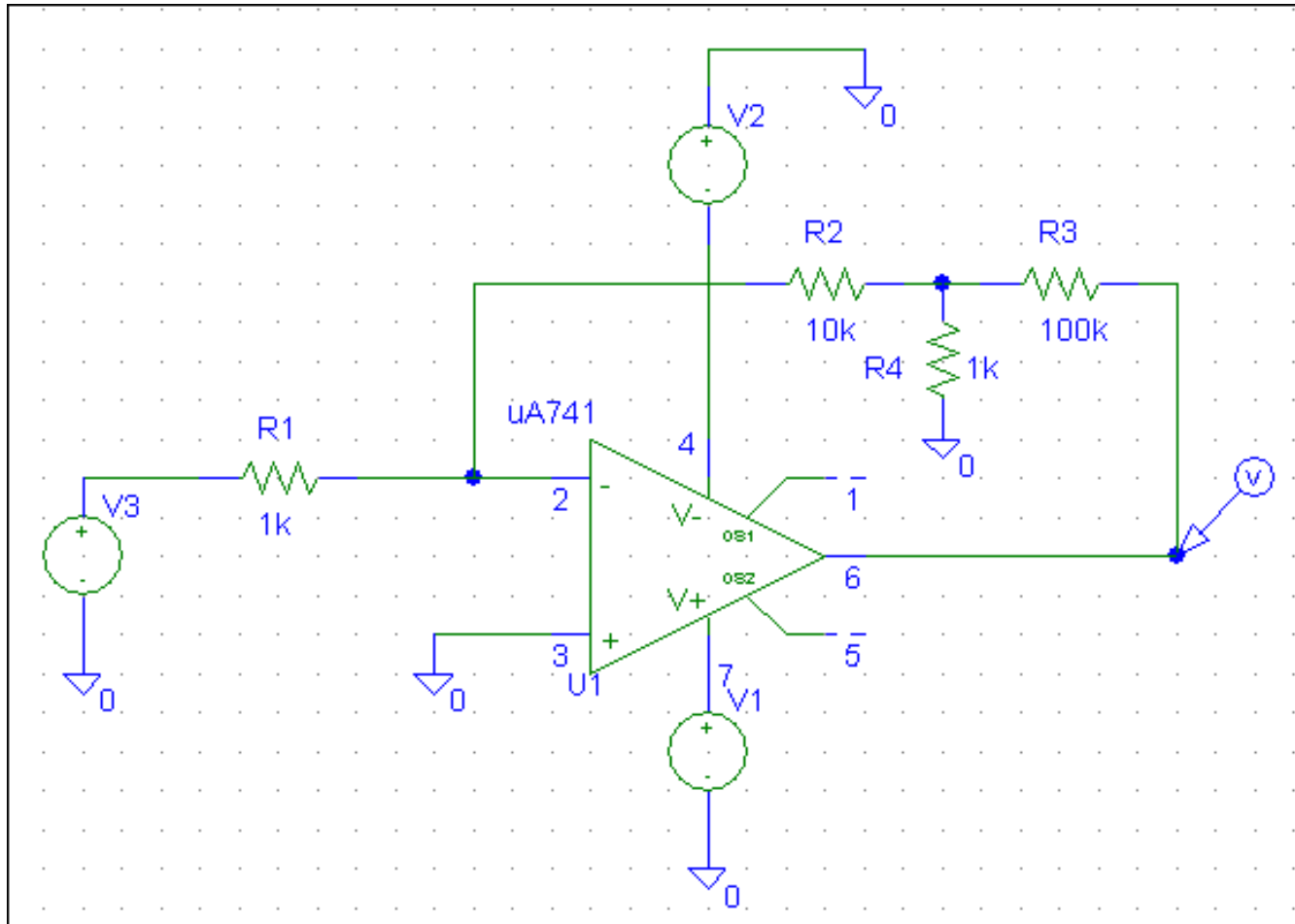
SIMULATIONS for high gain amplifier

SIM 5.3: $V_3(t)$, $V_O(t)$



SIMULATIONS for high gain amplifier

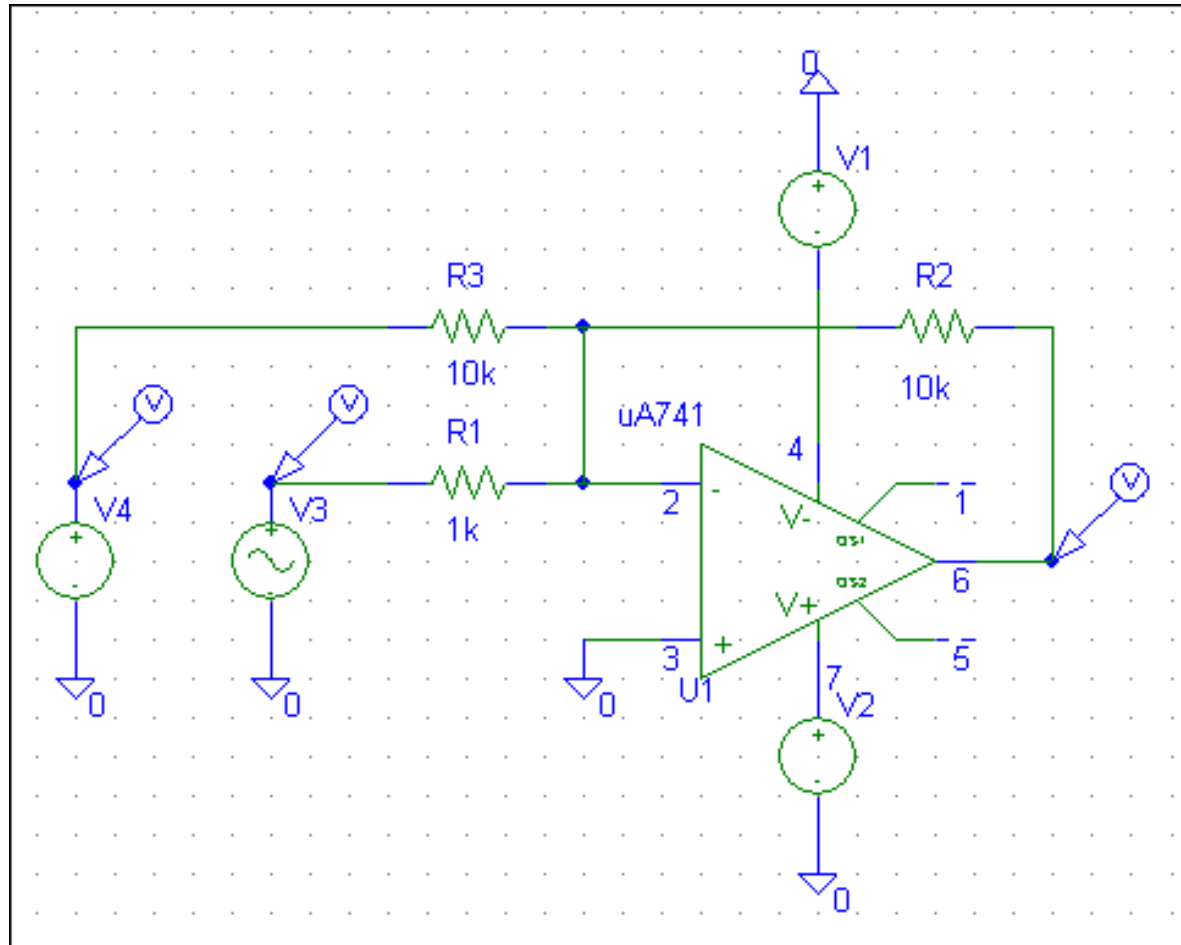
SIM 5.4: $V_O (V_3)$



SIMULATIONS for inverting voltage adder

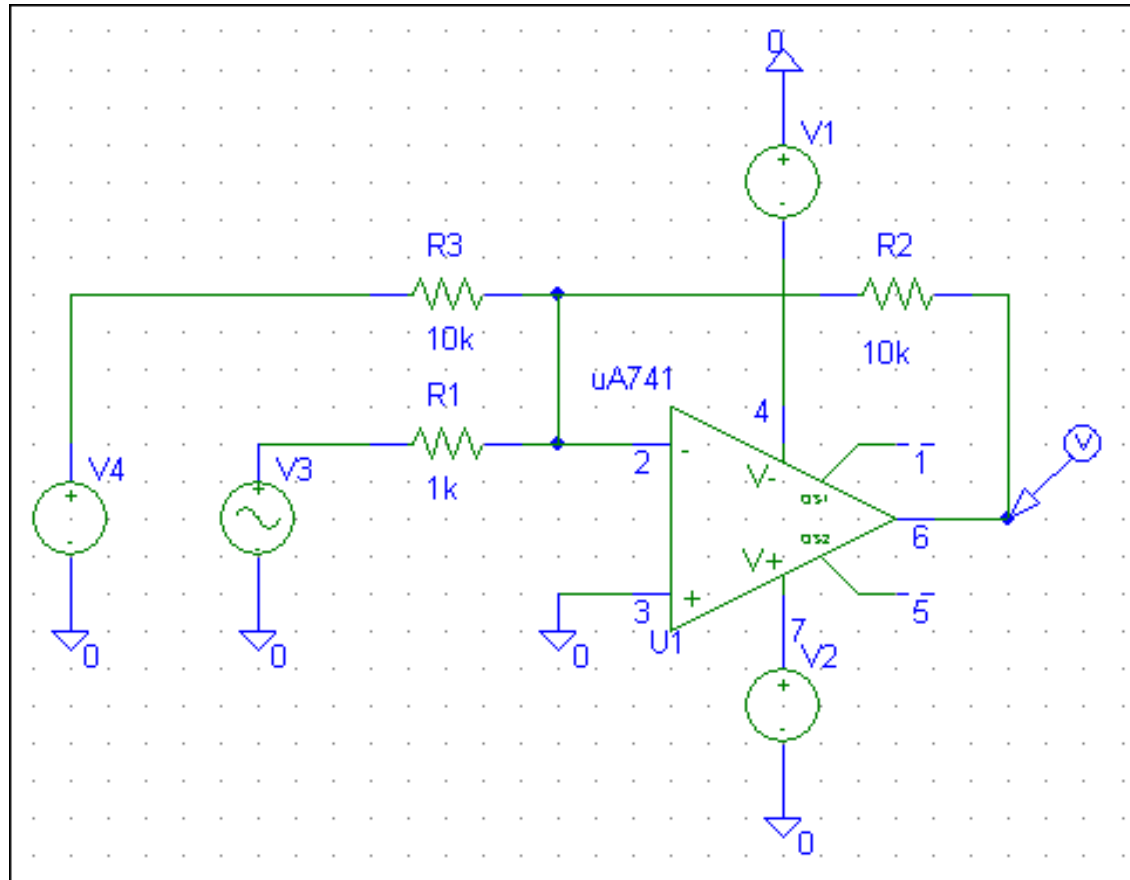
SIMULATIONS for inverting voltage adder

SIM 5.5: $V_3(t)$, $V_4(t)$, $V_O(t)$

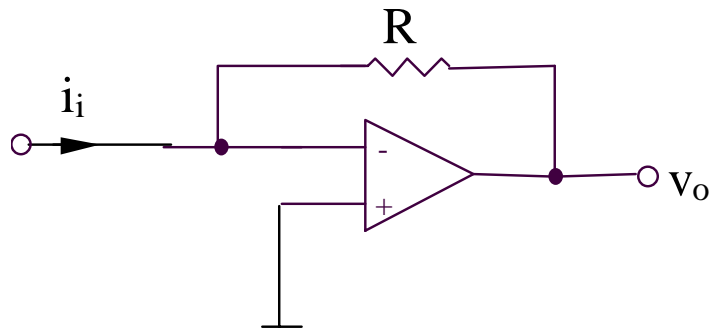


SIMULATIONS for inverting voltage adder

SIM 5.6: $V_O(t)$, V_4 - parameter

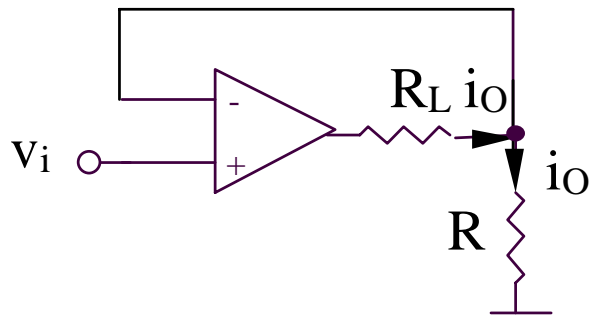


5.2.8. Current-voltage converter



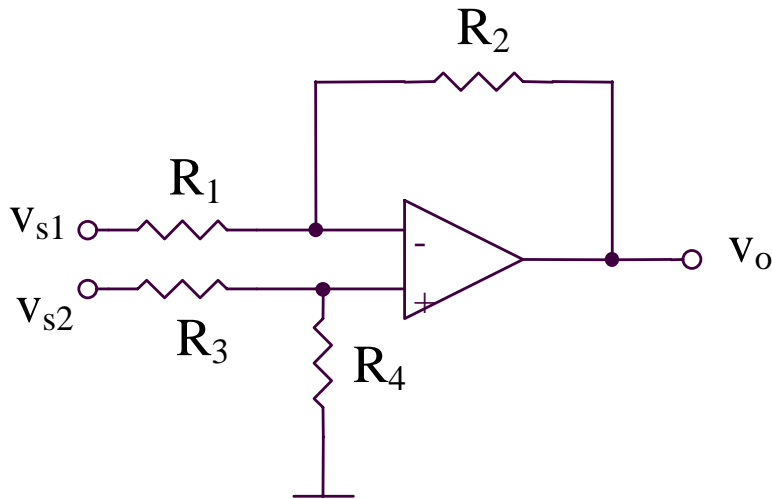
$$v_o = -Ri_i$$

5.2.9. Voltage-current converter



$$i_o = v_i / R$$

5.2.10. Difference circuit (1)



$$v_o = v_{s1} \left(-\frac{R_2}{R_1} \right) + v_{s2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_1} \right)$$

For obtain:

$$v_o = A(v_{s2} - v_{s1})$$

is necessary to impose the condition:

$$\frac{R_2}{R_1} = \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_1} \right)$$

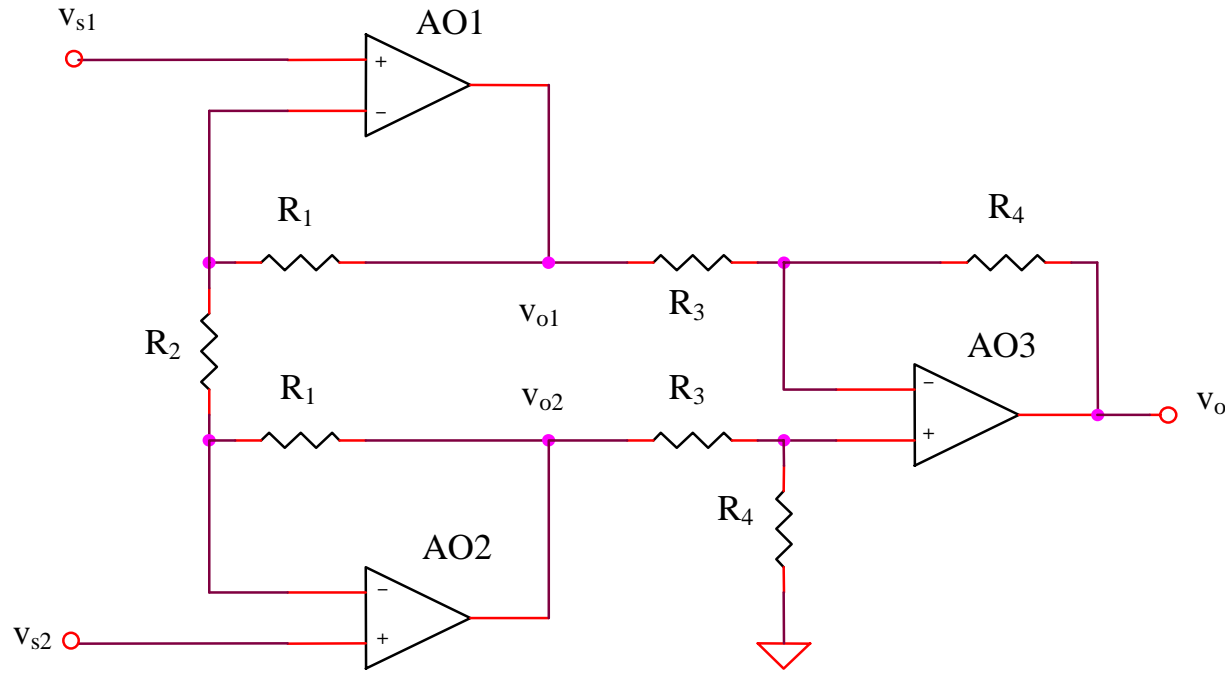
equivalent with:

$$R_1 R_4 = R_2 R_3$$

resulting:

$$v_o = \frac{R_2}{R_1} (v_{s2} - v_{s1})$$

5.2.11. Difference circuit (2)

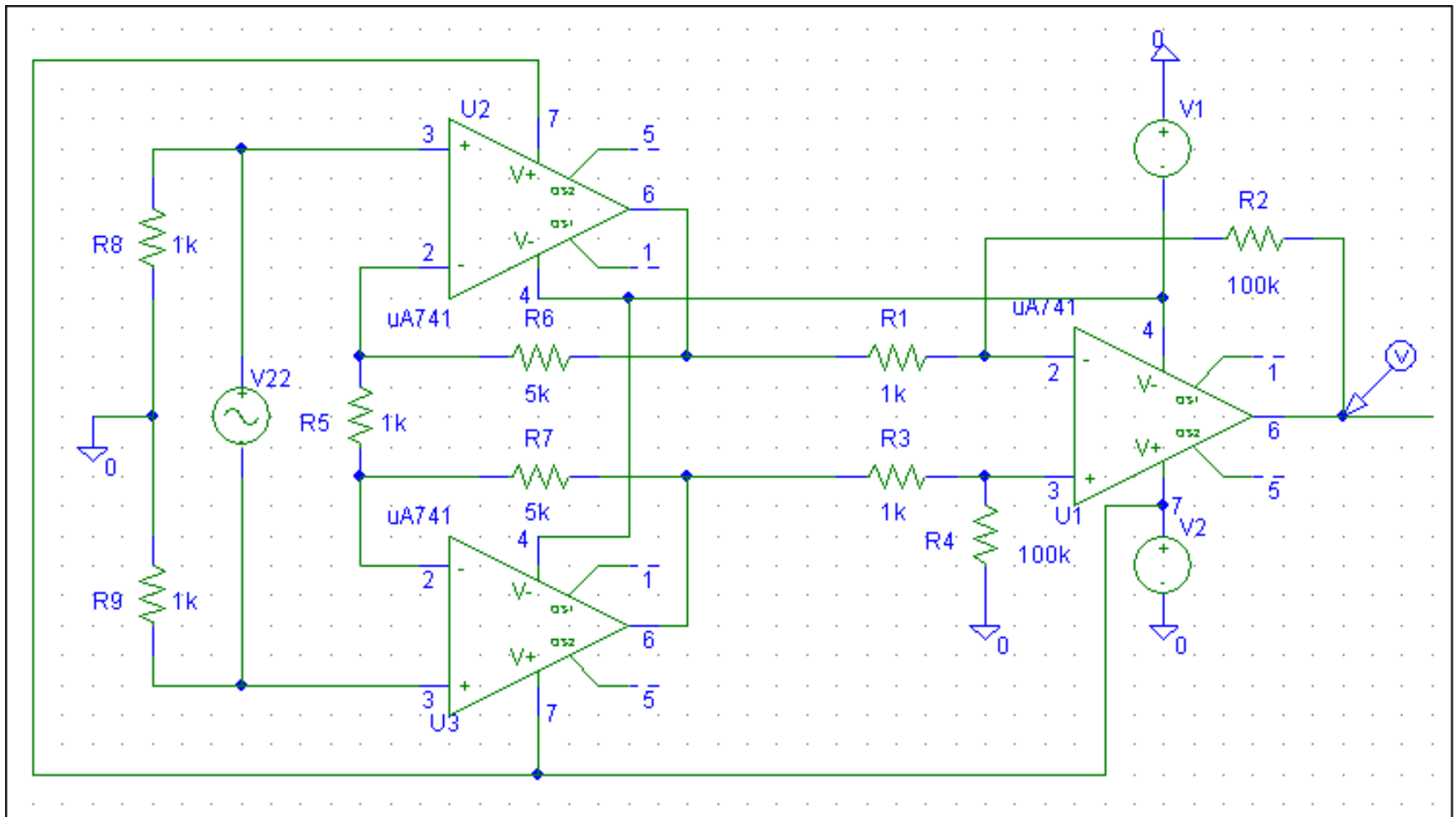


$$\left. \begin{aligned}
 v_{o1} &= v_{s1} \left(1 + \frac{R_1}{R_2} \right) - v_{s2} \frac{R_1}{R_2} \\
 v_{o2} &= v_{s2} \left(1 + \frac{R_1}{R_2} \right) - v_{s1} \frac{R_1}{R_2} \\
 v_o &= \frac{R_4}{R_3} (v_{o2} - v_{o1})
 \end{aligned} \right\} \Rightarrow A = \frac{v_o}{v_{s2} - v_{s1}} = \left(1 + 2 \frac{R_1}{R_2} \right) \frac{R_4}{R_3}$$

SIMULATIONS for difference circuit (2)

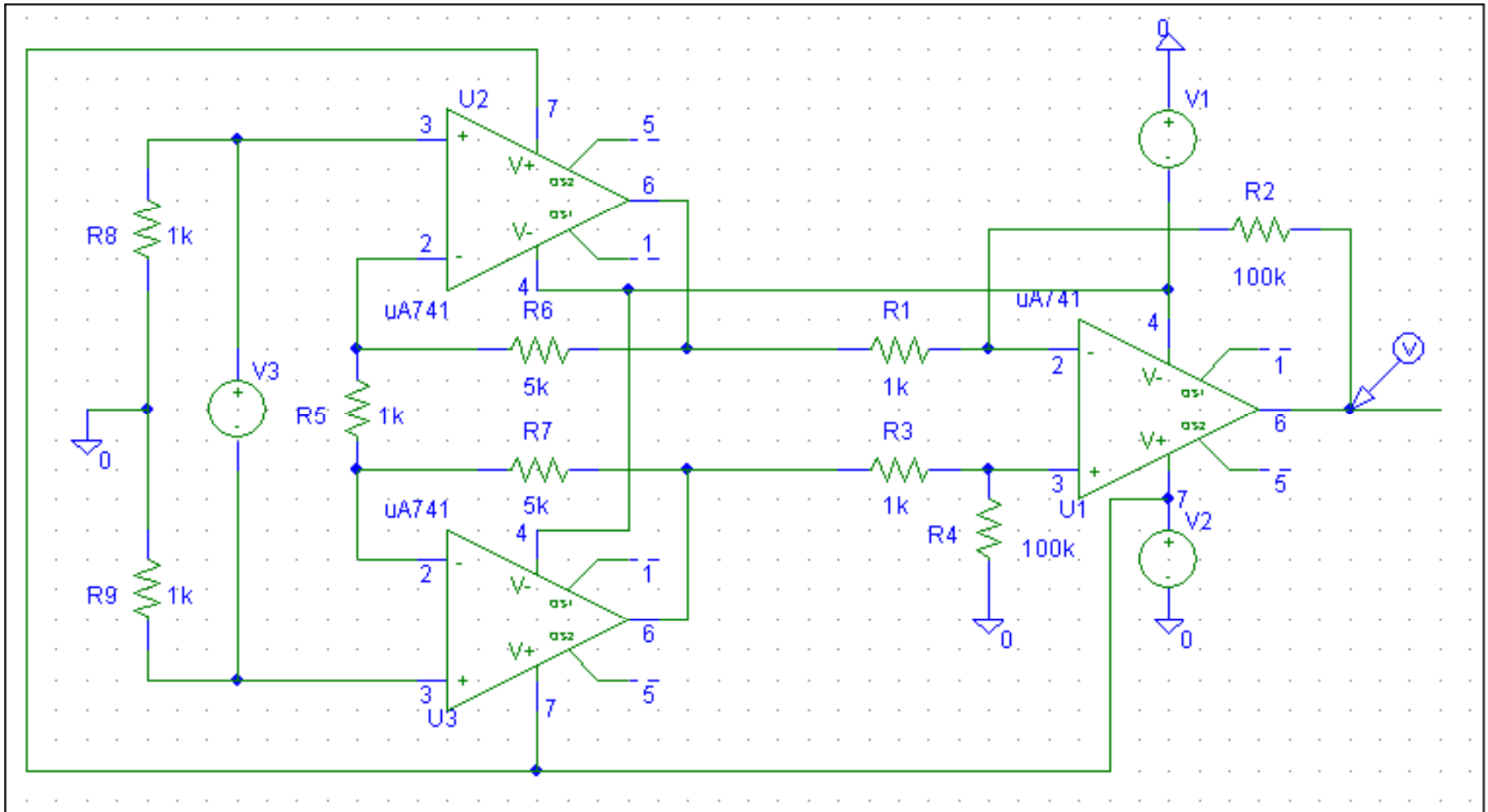
SIMULATIONS for difference circuit (2)

SIM 5.7: $V_O(t)$

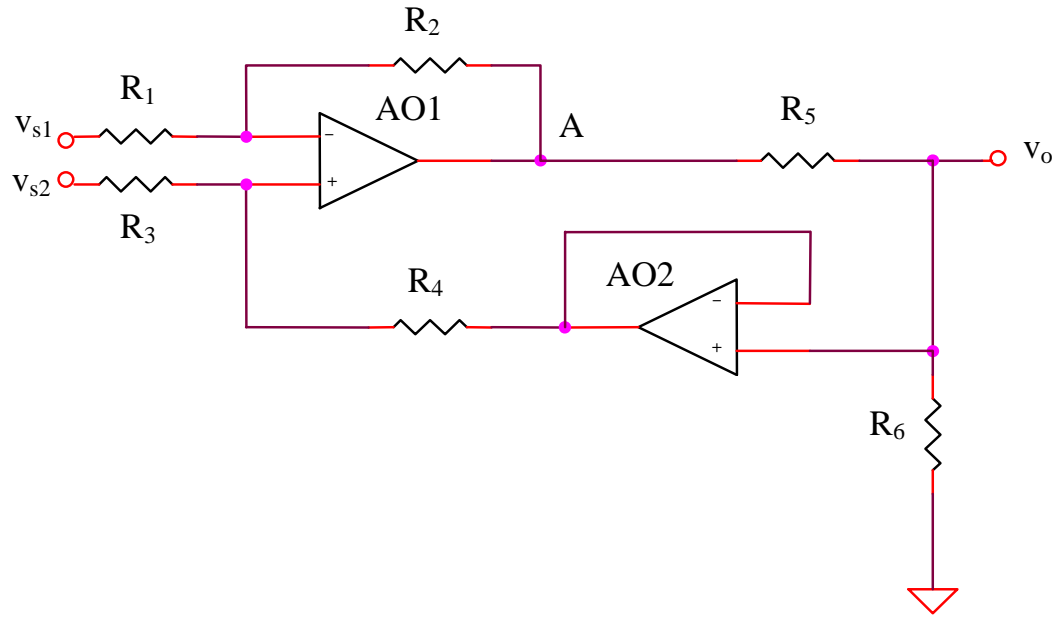


SIMULATIONS for difference circuit (2)

SIM 5.8: V_O (V_3)



5.2.12. Differential circuit (3)



$$v_A = v_{s1} \left(-\frac{R_2}{R_1} \right) + \frac{v_{s2} R_4 + v_o R_3}{R_3 + R_4} \left(1 + \frac{R_2}{R_1} \right)$$

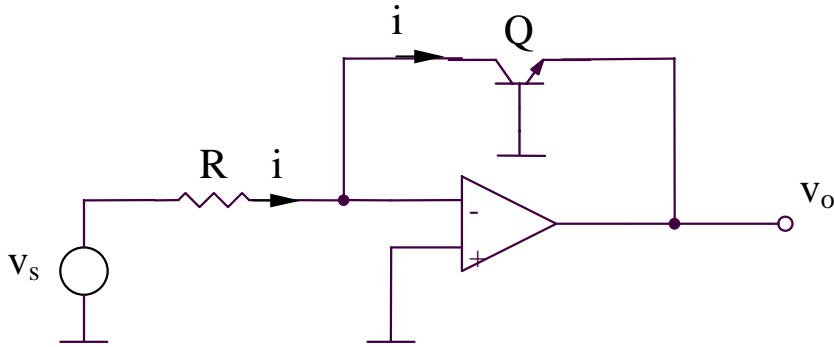
$$v_o = v_A \frac{R_6}{R_5 + R_6}$$

$$\Rightarrow v_o \left[\left(1 + \frac{R_5}{R_6} \right) - \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_4}{R_3}} \right] = v_{s2} \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_3}{R_4}} - v_{s1} \frac{R_2}{R_1}$$

$$v_o = A(v_{s2} - v_{s1})$$

$$\Rightarrow R_1 R_4 = R_2 R_3 \Rightarrow A = \frac{v_o}{v_{s2} - v_{s1}} = \frac{R_6 R_2}{R_5 R_1}$$

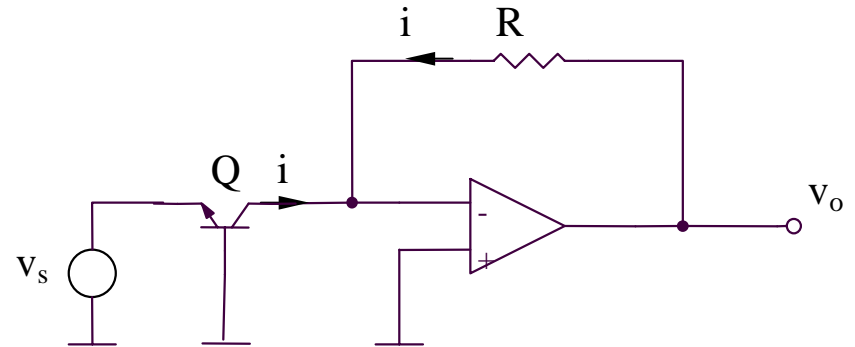
5.2.13. Logarithmic and anti-logarithmic converters



Logarithmic converter

$$v_o = -v_{BE} = -V_{th} \ln\left(\frac{i}{I_S}\right)$$

$$v_o = -V_{th} \ln\left(\frac{v_s}{RI_S}\right)$$



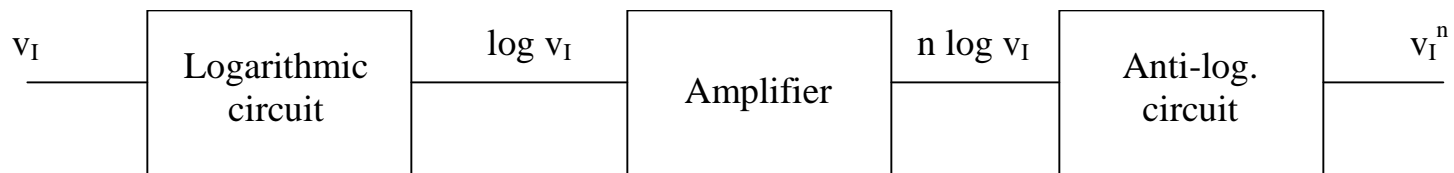
Anti-logarithmic converter

$$v_o = iR$$

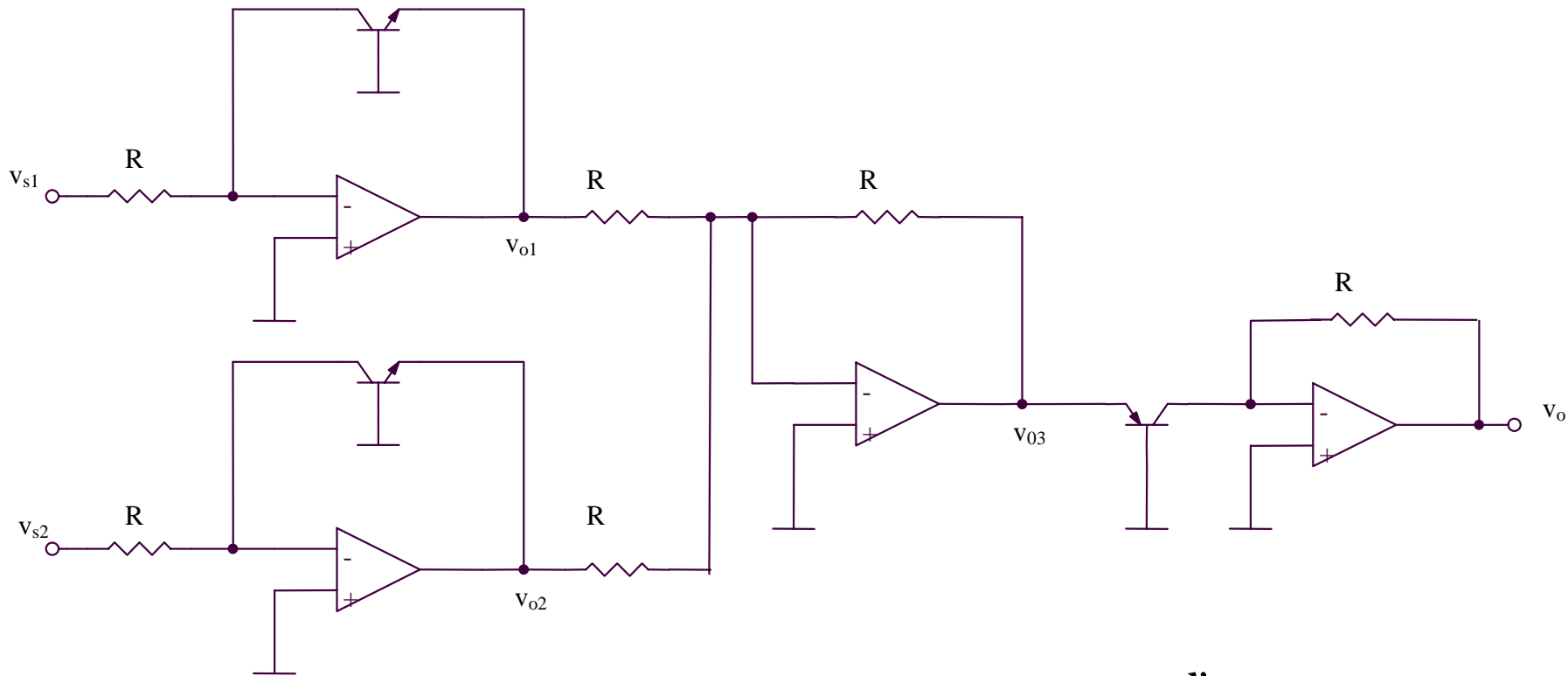
$$i = I_S e^{\frac{v_{BE}}{V_{th}}} = I_S e^{-\frac{v_s}{V_{th}}}$$

5.2.14. Circuit for the function $Y = X^n$

$$X^n = e^{n \ln x}$$



5.2.15. Multiplier circuit



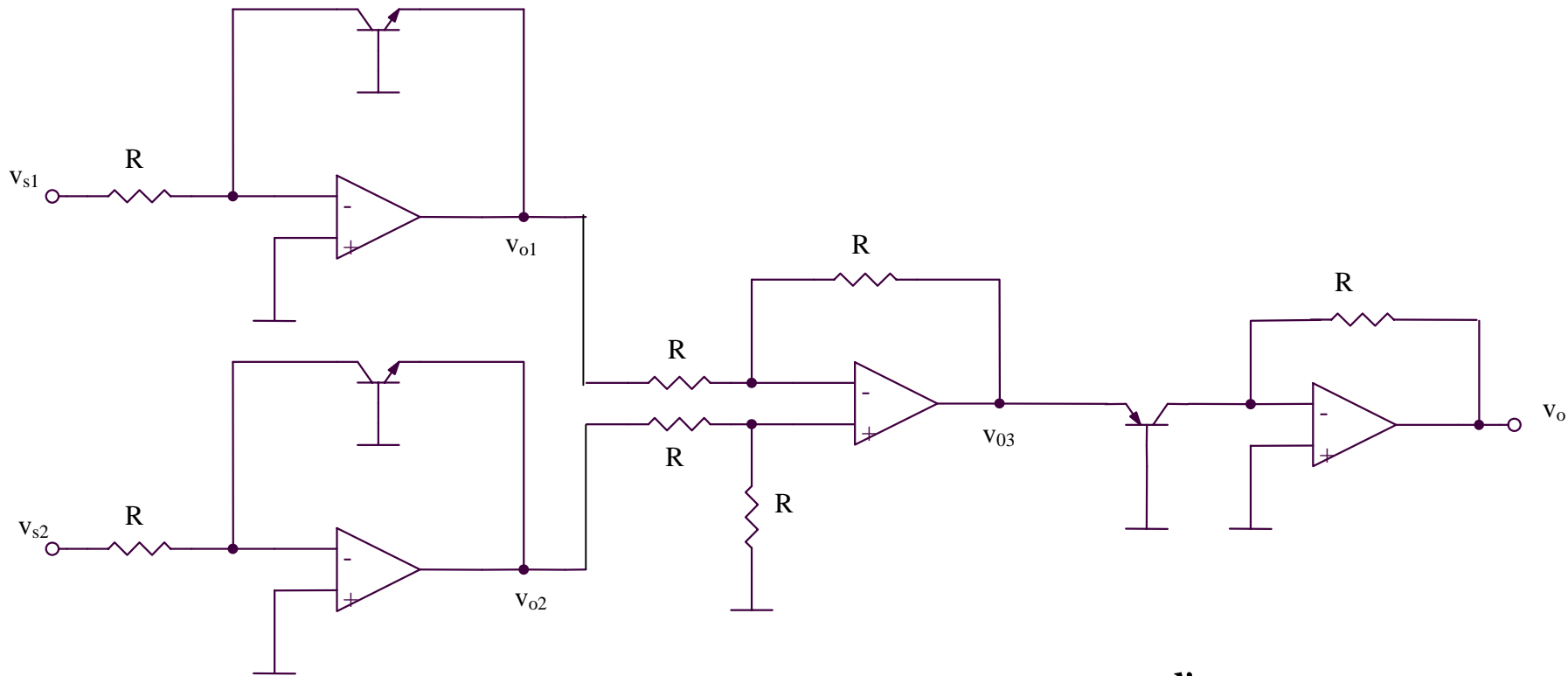
$$v_{o1} = -V_{th} \ln \frac{v_{s1}}{RI_S}$$

$$v_{o2} = -V_{th} \ln \frac{v_{s2}}{RI_S}$$

$$v_{o3} = \left(-\frac{R}{R}\right)v_{o1} + \left(-\frac{R}{R}\right)v_{o2} = -(v_{o1} + v_{o2}) = V_{th} \ln \frac{v_{s1}v_{s2}}{R^2 I_S^2}$$

$$v_o = -RI_S e^{\frac{v_{o3}}{V_{th}}} = -\frac{v_{s1}v_{s2}}{RI_S}$$

5.2.16. Divider circuit



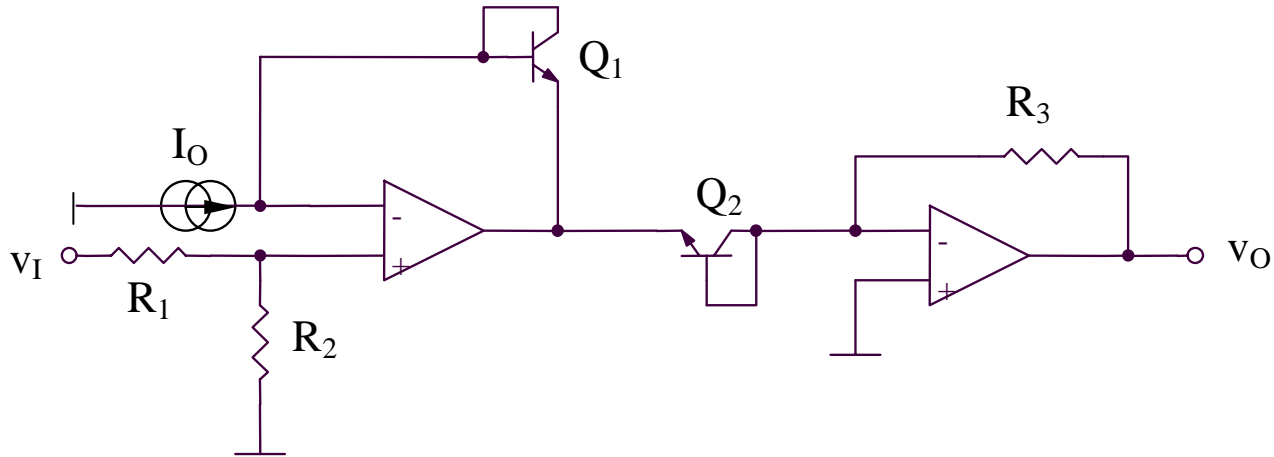
$$v_{o1} = -V_{th} \ln \frac{v_{s1}}{RI_S}$$

$$v_{o2} = -V_{th} \ln \frac{v_{s2}}{RI_S}$$

$$v_{o3} = v_{o2} - v_{o1} = V_{th} \ln \frac{v_{s1}}{v_{s2}}$$

$$v_o = -RI_S e^{v_{o3}/V_{th}} = -RI_S \frac{v_{s1}}{v_{s2}}$$

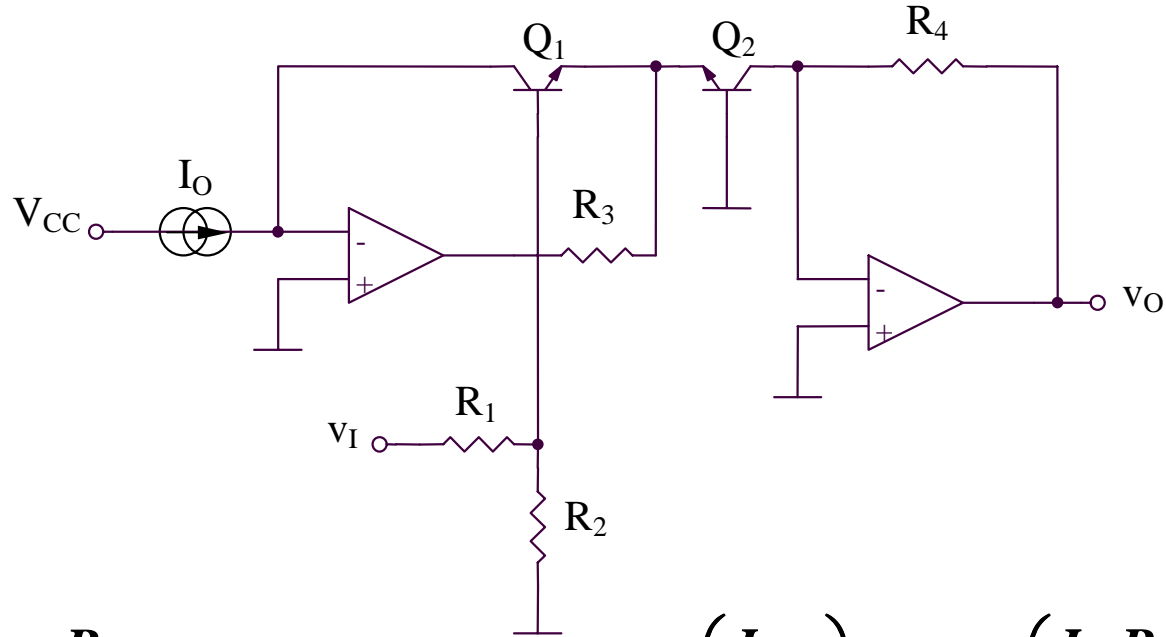
5.2.17. Exponential circuit (1)



$$v_I \frac{R_2}{R_1 + R_2} = v_{BE1} - v_{BE2} = V_{th} \ln\left(\frac{I_{C1}}{I_{C2}}\right) = V_{th} \ln\left(\frac{I_0 R_3}{v_O}\right) \Rightarrow$$

$$\Rightarrow v_O = I_0 R_3 e^{-\frac{v_I R_2}{V_{th} R_1 + R_2}}$$

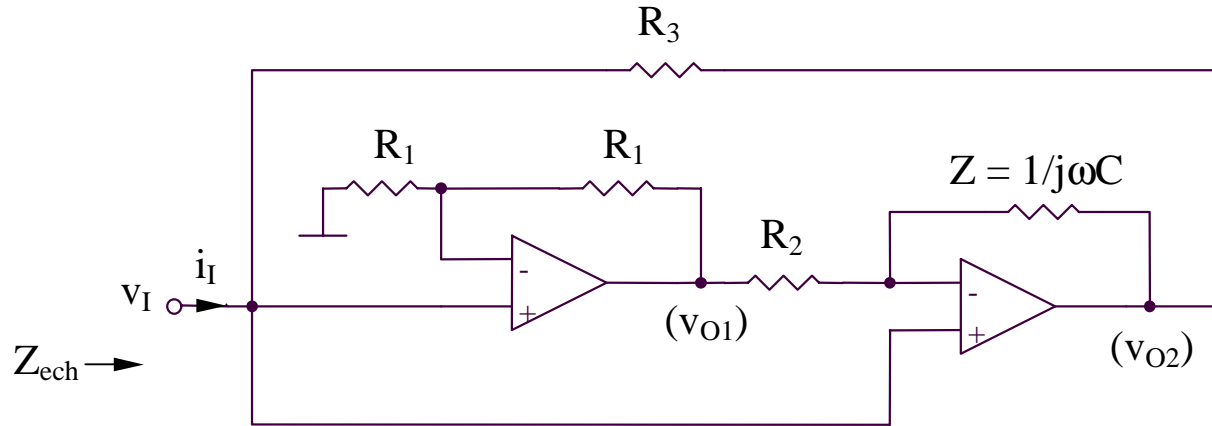
5.2.18. Exponential circuit (2)



$$v_I \frac{R_2}{R_1 + R_2} = v_{BE1} - v_{BE2} = V_{th} \ln \left(\frac{I_{C1}}{I_{C2}} \right) = V_{th} \ln \left(\frac{I_O R_4}{v_O} \right) \Rightarrow$$

$$\Rightarrow v_O = I_O R_4 e^{-\frac{v_I}{V_{th}} \frac{R_2}{R_1 + R_2}}$$

5.2.19. Inductance simulator



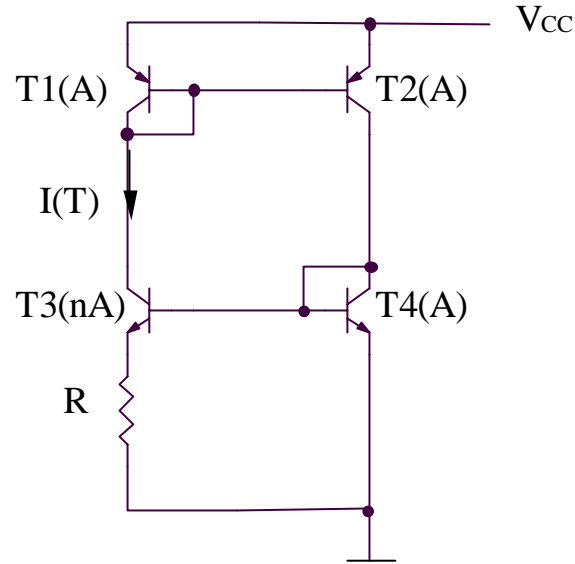
$$\left. \begin{aligned} v_{O2} &= v_I \left(1 + \frac{Z}{R_2} \right) + v_{O1} \left(-\frac{Z}{R_2} \right) \\ v_{O1} &= 2v_I \end{aligned} \right\} \Rightarrow v_{O2} = v_I \left(1 - \frac{Z}{R_2} \right)$$

$$\left. \begin{aligned} v_{O2} &= v_I \left(1 - \frac{Z}{R_2} \right) \\ i_I &= \frac{v_I - v_{O2}}{R_3} \end{aligned} \right\} \Rightarrow i_I = v_I \frac{Z}{R_2 R_3} \Rightarrow$$

$$\Rightarrow Z_{ech} = \frac{v_I}{i_I} = \frac{R_2 R_3}{Z} = j\omega(R_2 R_3 C) = j\omega L_{ech}$$

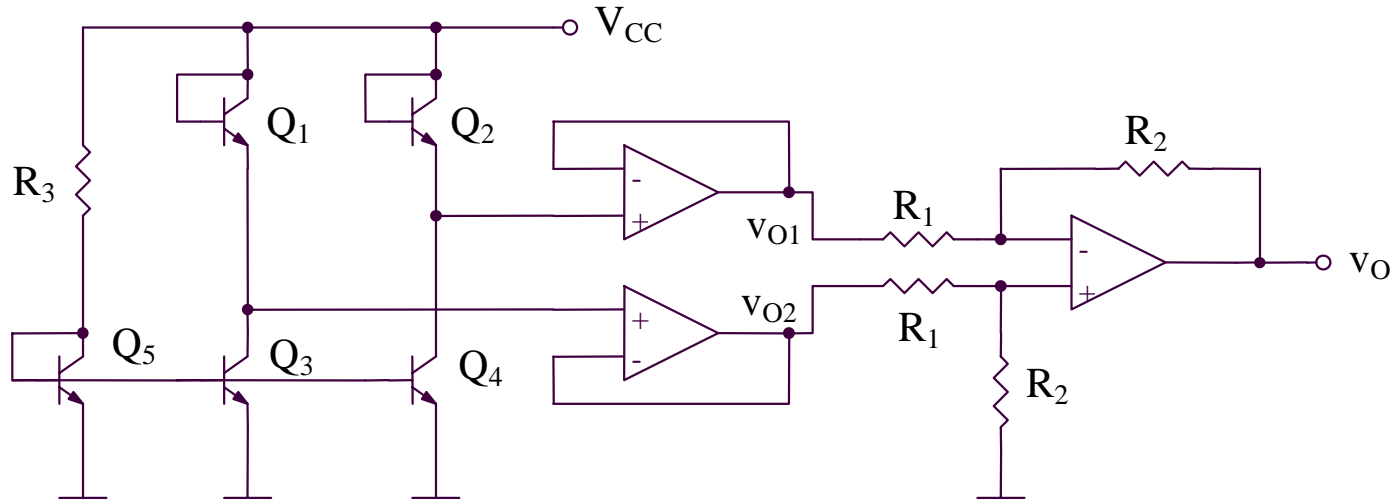
$$L_{ech} = R_2 R_3 C$$

5.2.20. Temperature sensors



$$I(T) = \frac{V_{BE4} - V_{BE3}}{R} = \frac{V_{th}}{R} \ln \left(\frac{I_{C4} I_{S3}}{I_{C3} I_{S4}} \right) = \frac{V_{th}}{R} \ln n$$

5.2.20. Temperature sensors

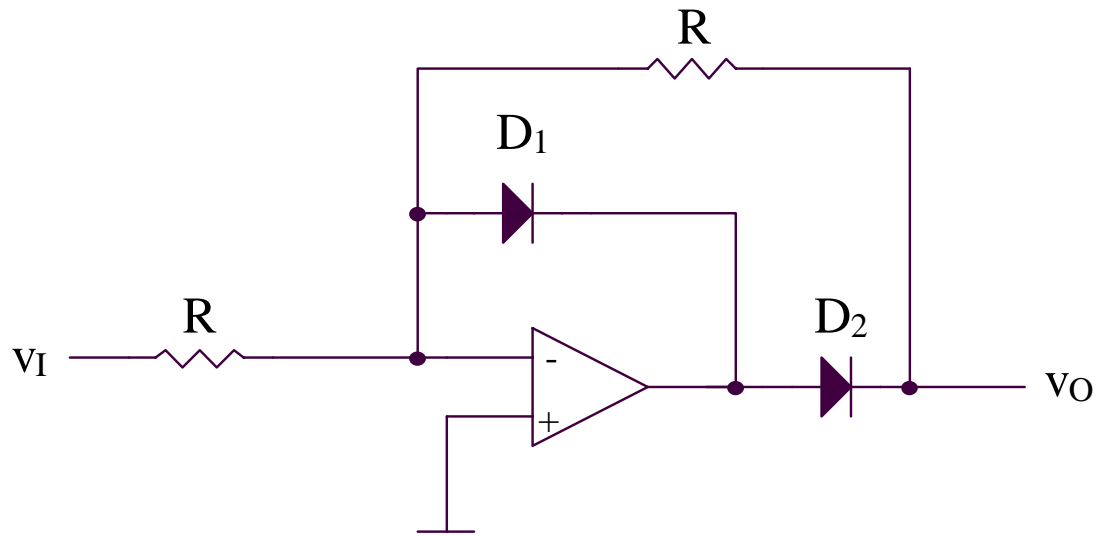


$$v_O = \frac{R_2}{R_1} (v_{O2} - v_{O1}) = \frac{R_2}{R_1} (v_{BE2} - v_{BE1}) = \frac{R_2}{R_1} V_{th} \ln \left(\frac{I_{C2} I_{S1}}{I_{C1} I_{S2}} \right) \Rightarrow$$

$$\Rightarrow v_O = \frac{R_2}{R_1} V_{th} \ln \left(\frac{I_{C4} I_{S1}}{I_{C3} I_{S2}} \right) = \frac{R_2}{R_1} V_{th} \ln \left(\frac{I_{S4} I_{S1}}{I_{S3} I_{S2}} \right) = \frac{R_2}{R_1} V_{th} \ln \left(\frac{A_4 A_1}{A_3 A_2} \right) = MT$$

$$M = \frac{R_2 k}{R_1 q} \ln \left(\frac{A_4 A_1}{A_3 A_2} \right)$$

5.2.21. Half-wave rectifier

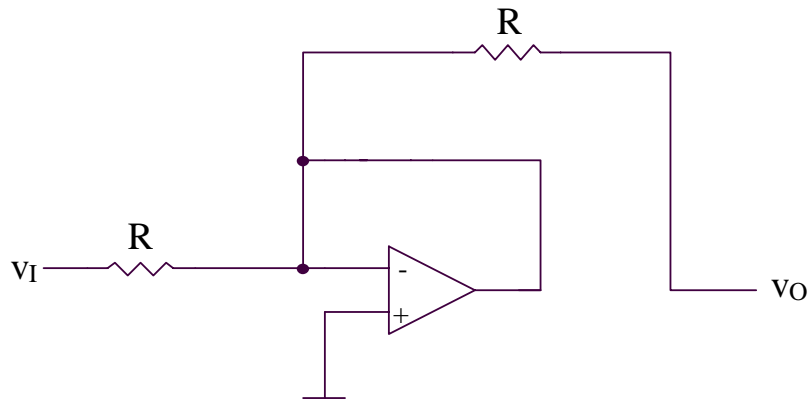


$v_I > 0 \Rightarrow v_{O1} < 0 \Rightarrow D_1 \text{ open}, D_2 \text{ blocked}$

$v_I < 0 \Rightarrow v_{O1} > 0 \Rightarrow D_2 \text{ open}, D_1 \text{ blocked}$

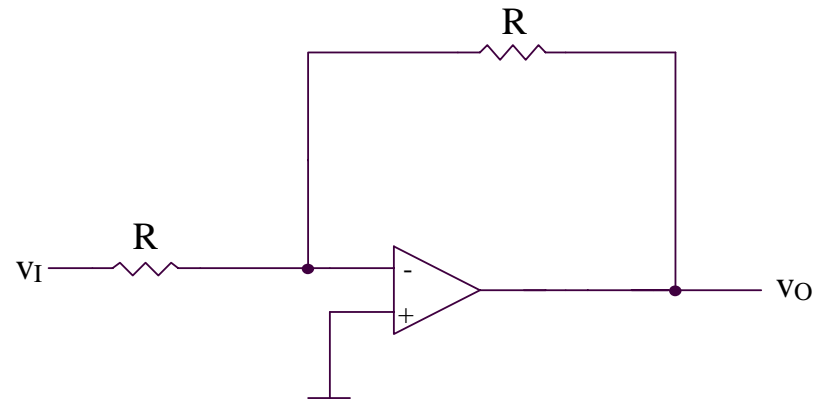
5.2.21. Half-wave rectifier

$$v_I > 0$$



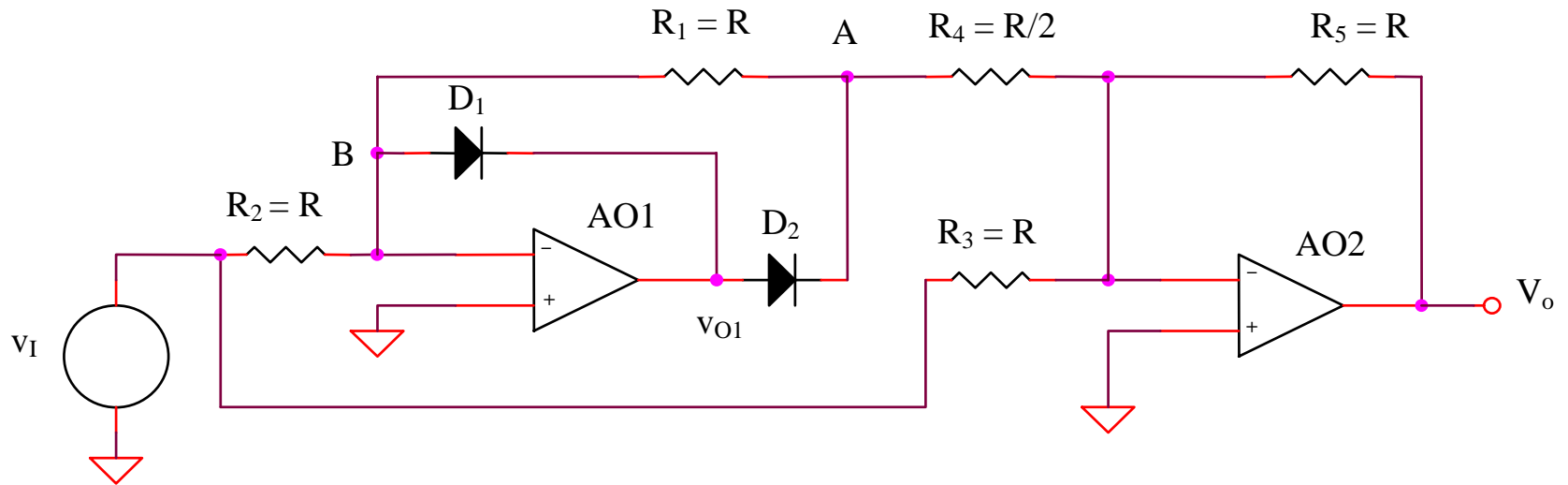
$$v_O = 0$$

$$v_I < 0$$



$$v_O = -\frac{R}{R}v_I = -v_I$$

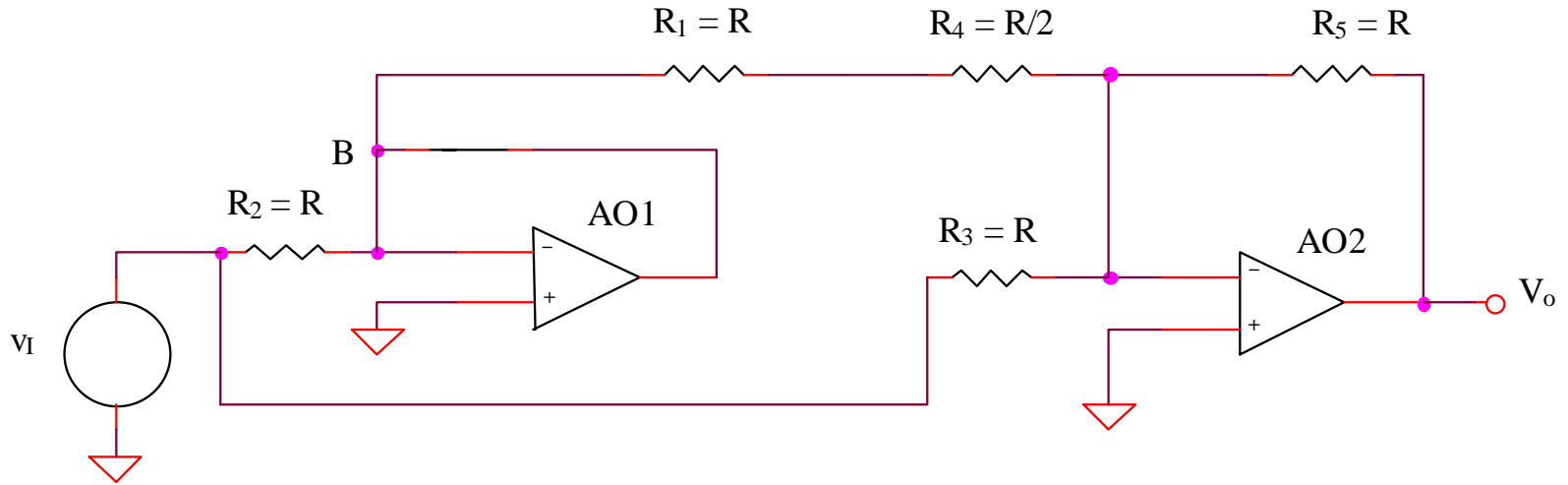
5.2.22. Full-wave rectifier (1)



$v_I > 0 \Rightarrow v_{O1} < 0 \Rightarrow D_1 \text{ open}, D_2 \text{ blocked}$

$v_I < 0 \Rightarrow v_{O1} > 0 \Rightarrow D_2 \text{ open}, D_1 \text{ blocked}$

5.2.22. Full-wave rectifier (1)

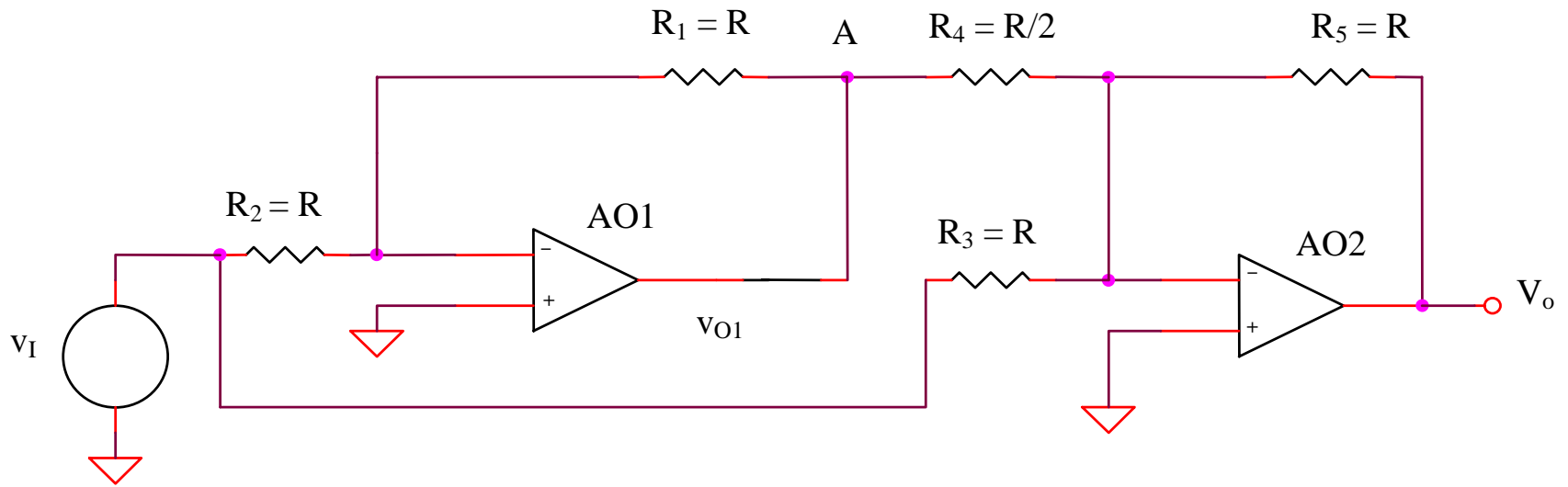


$$v_I > 0$$

$$V_B = 0$$

$$v_O = -\frac{R_5}{R_3} v_I = -v_I$$

5.2.22. Full-wave rectifier (1)



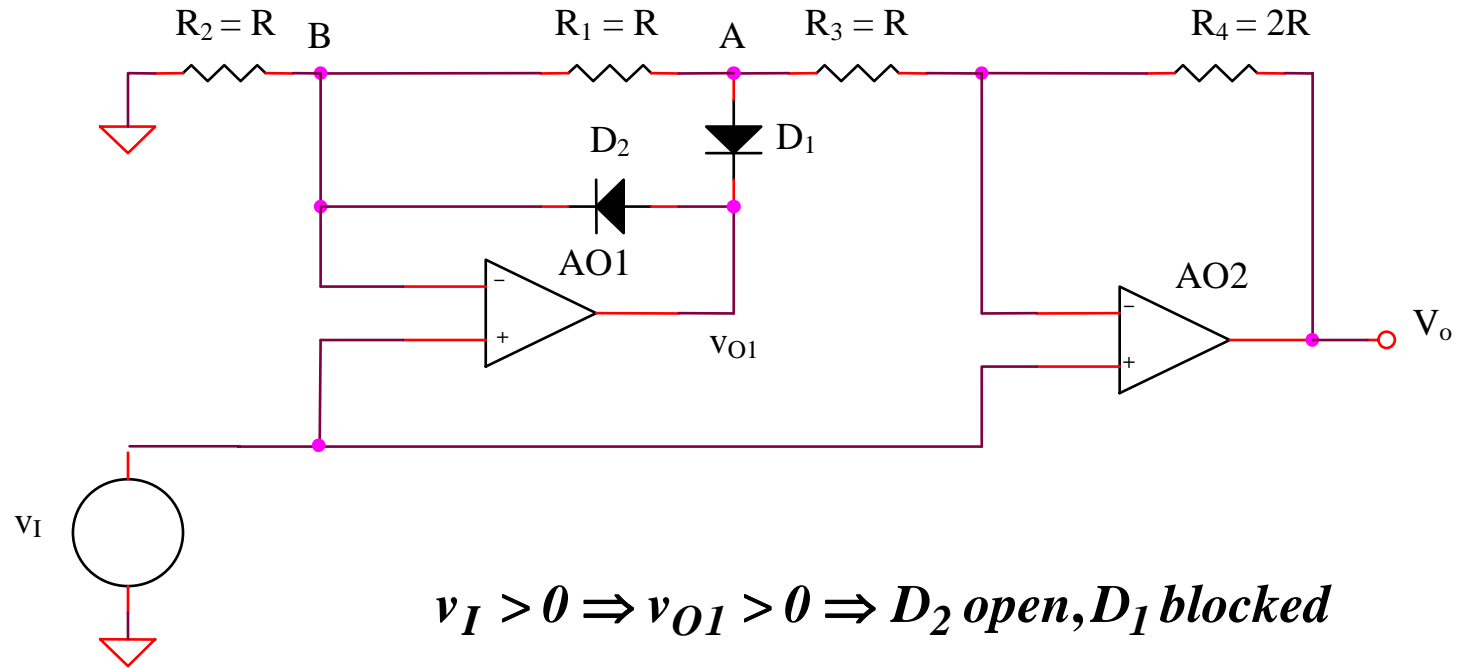
$$v_I < 0$$

$$V_A = -\frac{R_1}{R_2} v_I$$

$$v_O = -\frac{R_5}{R_4} V_A - \frac{R_5}{R_3} v_I = v_I$$

Conclusion: $v_O = -|v_I|$

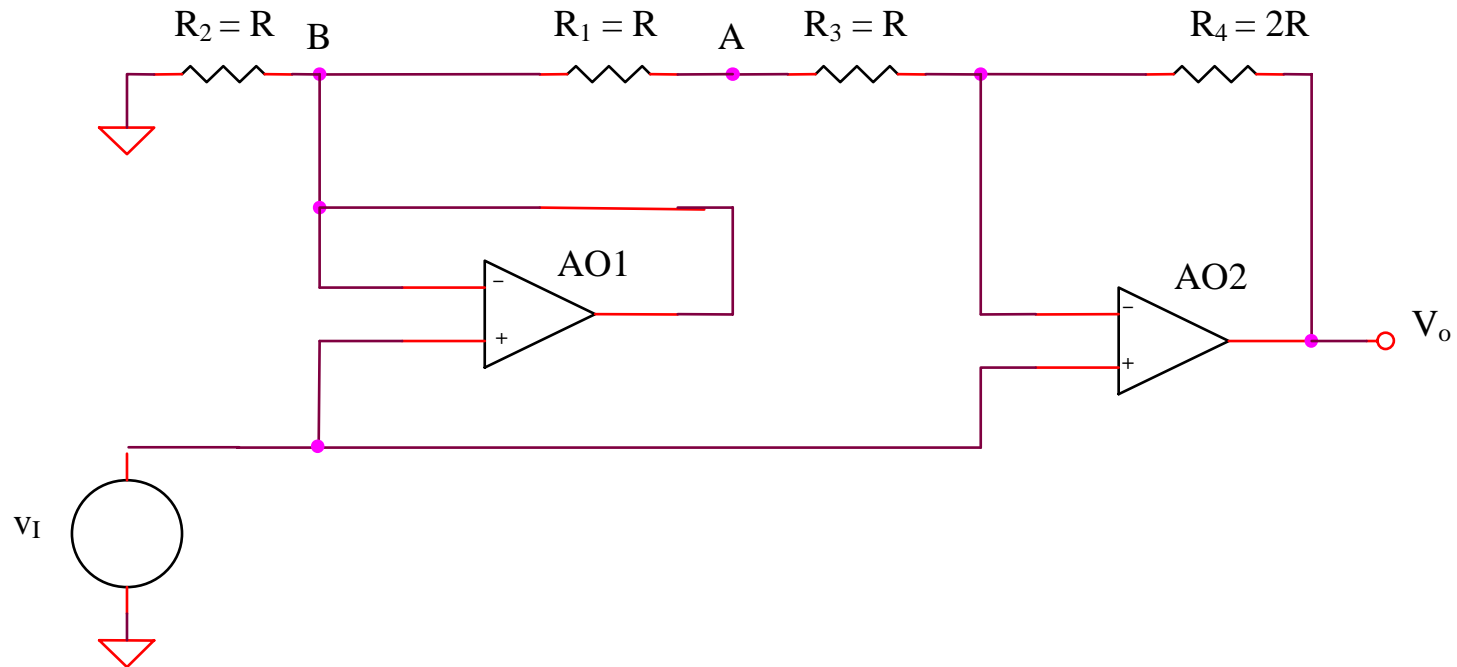
5.2.23. Full-wave rectifier (2)



$v_I > 0 \Rightarrow v_{O1} > 0 \Rightarrow D_2 \text{ open}, D_1 \text{ blocked}$

$v_I < 0 \Rightarrow v_{O1} < 0 \Rightarrow D_1 \text{ open}, D_2 \text{ blocked}$

5.2.23. Full-wave rectifier (2)

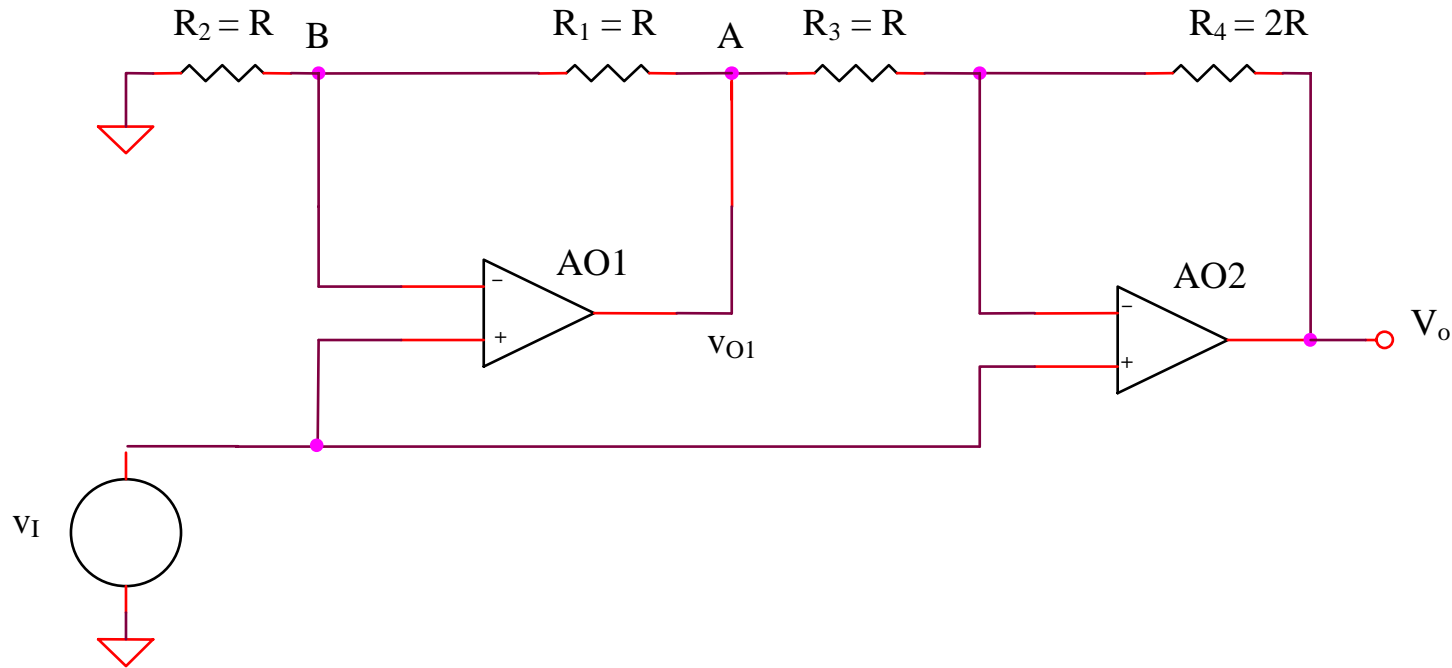


$$v_I > 0$$

$$V_B = v_I$$

$$v_O = \left(1 + \frac{R_4}{R_1 + R_3} \right) v_I - \frac{R_4}{R_1 + R_3} V_B = v_I$$

5.2.23. Full-wave rectifier (2)



$$v_I < 0$$

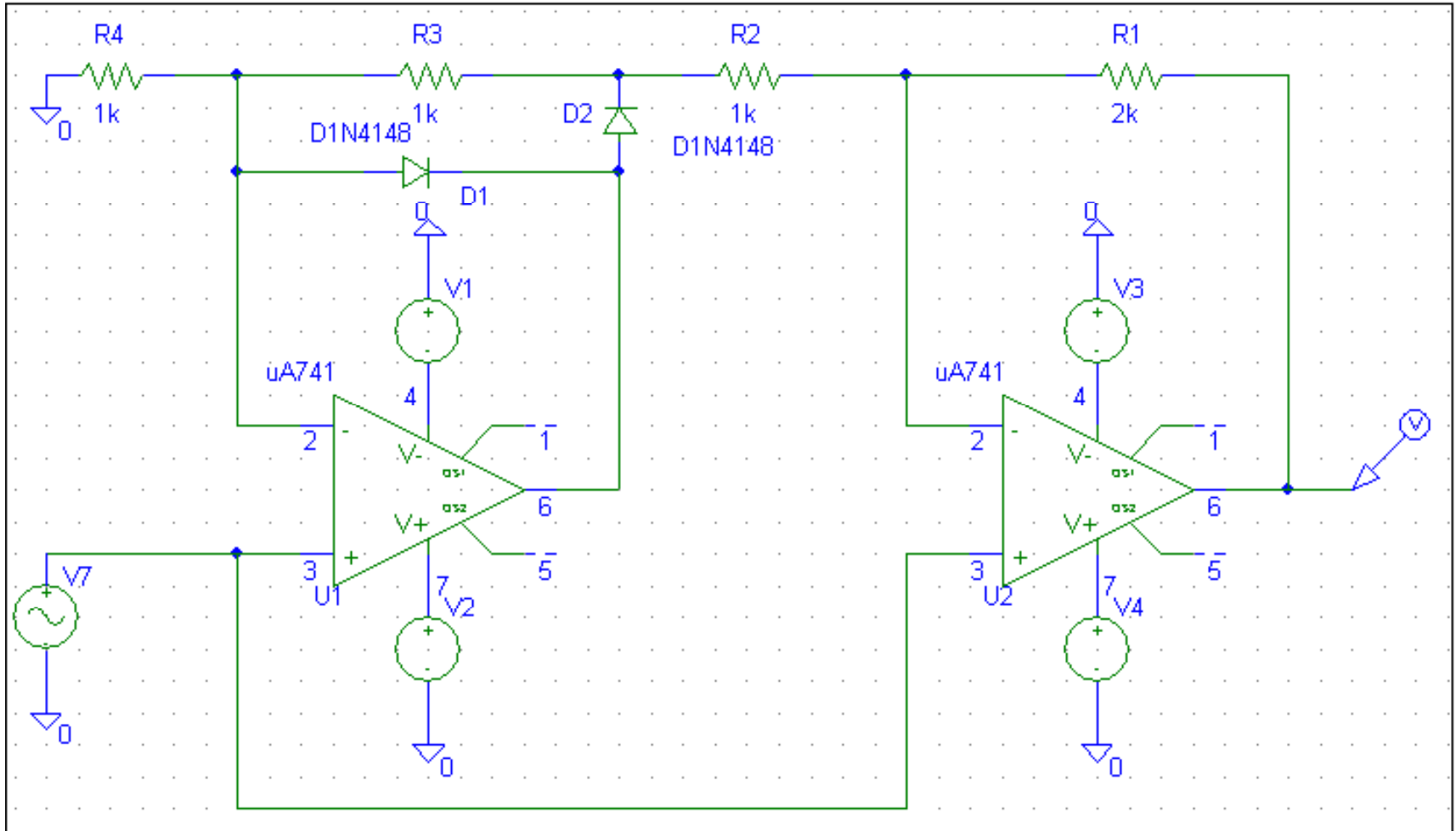
$$V_A = \left(1 + \frac{R_1}{R_2}\right)v_I = 2v_I \quad v_O = \left(1 + \frac{R_4}{R_3}\right)v_I - \frac{R_4}{R_3}V_A = -v_I$$

Conclusion: $v_O = |v_I|$

SIMULATIONS for full-wave rectifier (2)

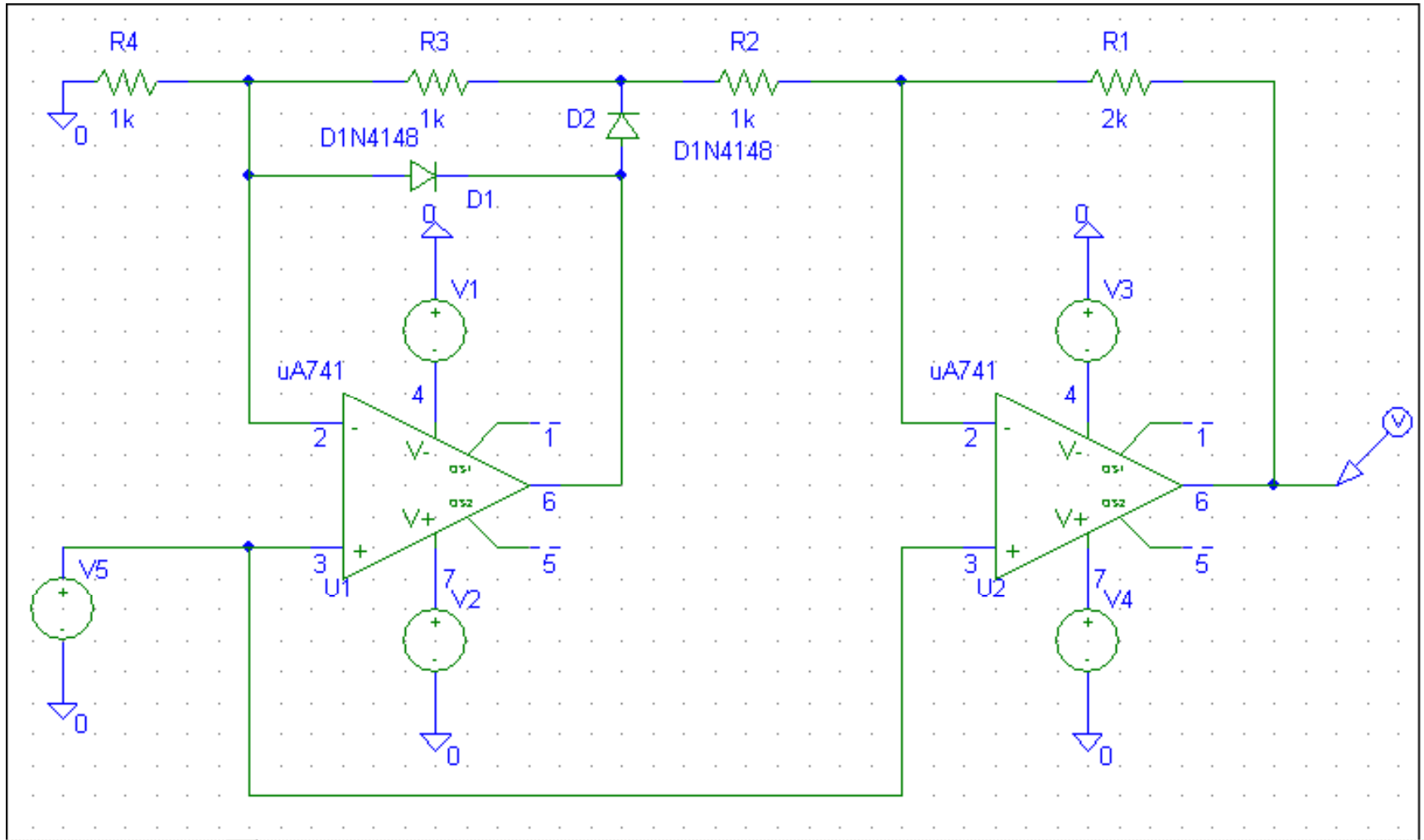
SIMULATIONS for full-wave rectifier (2)

SIM 5.9: $V_O(t)$

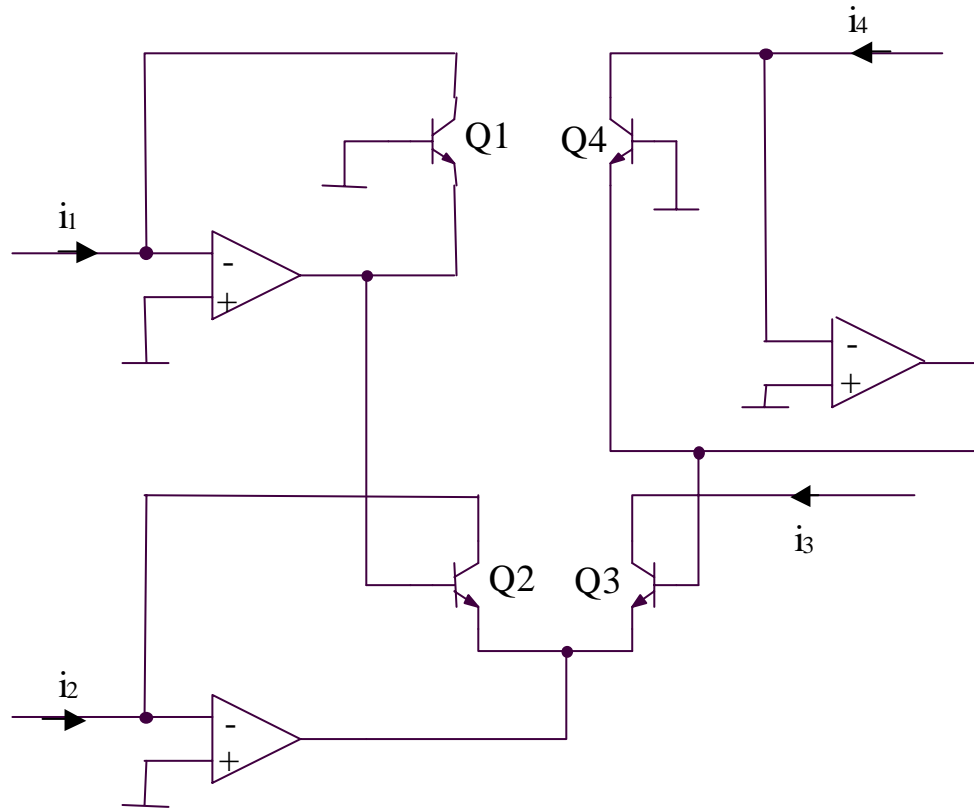


SIMULATIONS for full-wave rectifier (2)

SIM 5.10: V_O (V_5)



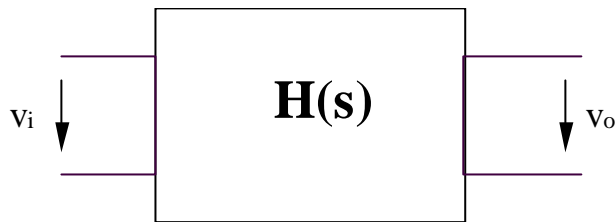
5.2.24. Multifunctional circuit



$$V_{BE1} + V_{BE2} = V_{BE3} + V_{BE4}$$

$$V_{th} \ln \frac{i_1}{I_S} + V_{th} \ln \frac{i_2}{I_S} = V_{th} \ln \frac{i_3}{I_S} + V_{th} \ln \frac{i_4}{I_S} \Rightarrow i_1 i_2 = i_3 i_4$$

5.2.25. Filters



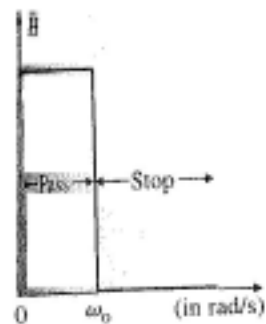
Classification

- **Low-pass filters**
- **High-pass filters**
- **Band-pass filters**
- **Band-reject filters**

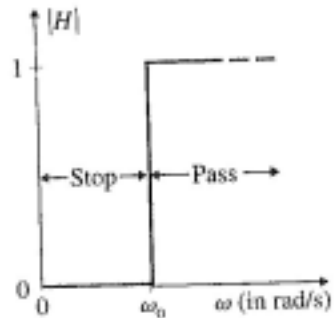
$$H(s) = \frac{a_m s^m + \dots + a_2 s^2 + a_1 s + a_0}{s^n + \dots + b_2 s^2 + b_1 s + b_0}, n \geq m$$

5.2.25. Filters

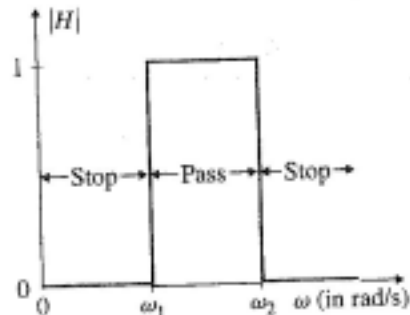
Ideal filter characteristics



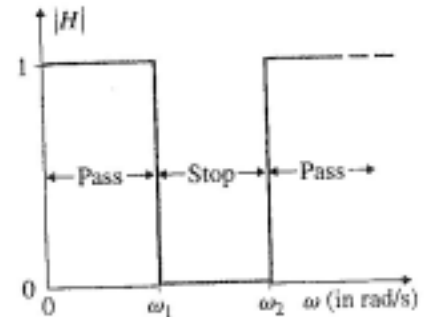
(a) Ideal low-pass filter



(b) Ideal high-pass filter

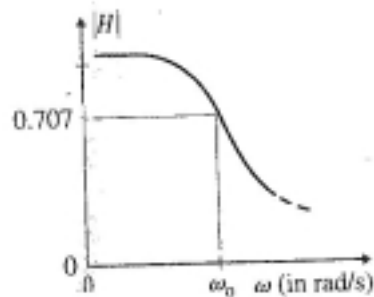


(c) Ideal band-pass filter

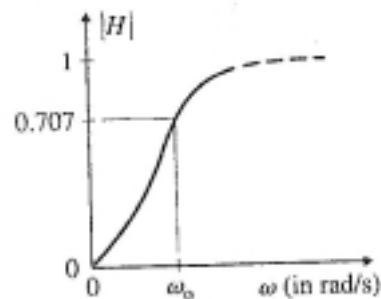


(d) Ideal band-reject filter

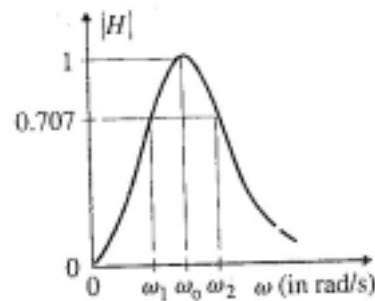
Realistic filter characteristics



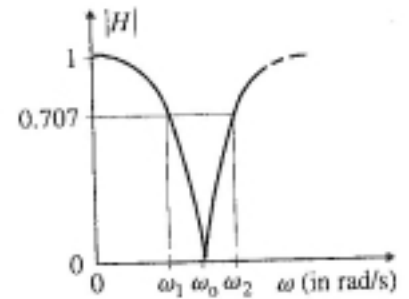
(a) Low-pass filter



(b) High-pass filter



(c) Band-pass filter

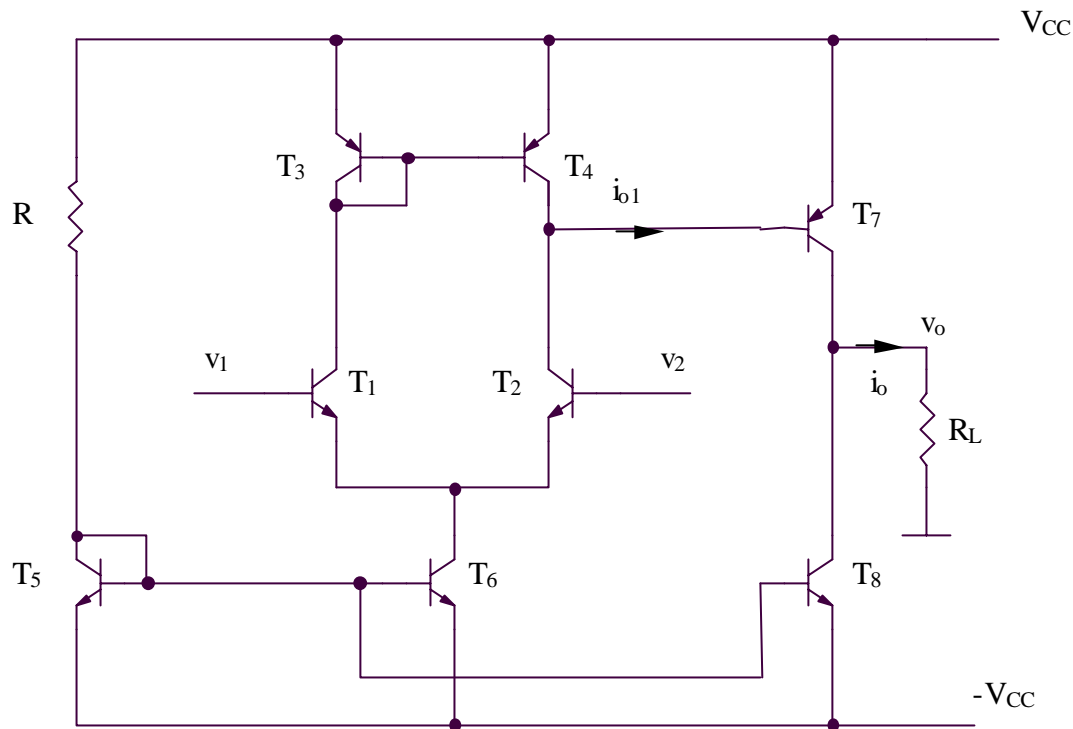


(d) Band-reject filter

5.3. Study of bipolar operational amplifier structures

5.3.1. Study of an operational amplifier with two stages

5.3.1. Study of an operational amplifier with two stages



Static regime

$$I_{C5,6,7,8} = \frac{2V_{CC} - V_{BE}}{R}$$

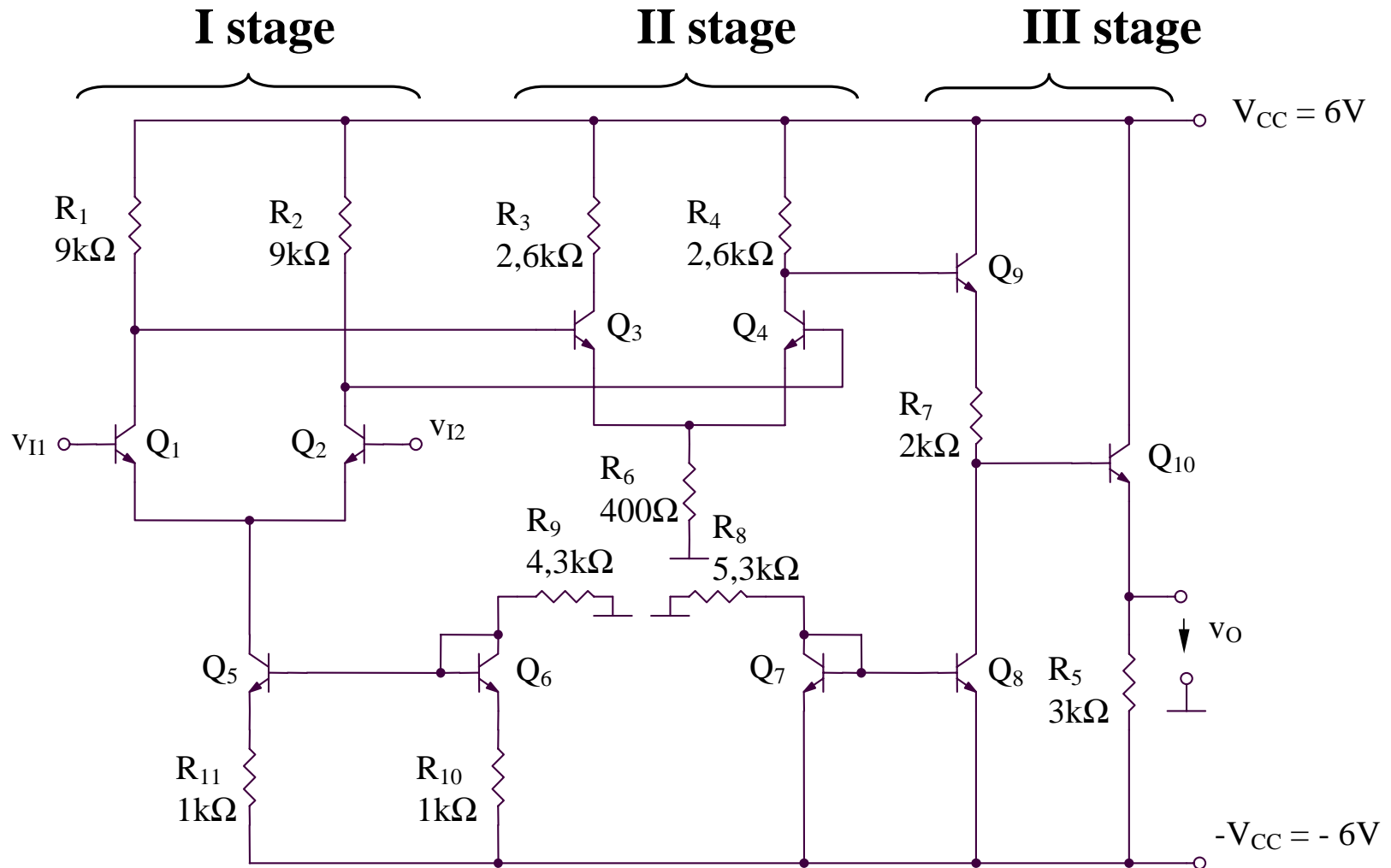
$$I_{C1,2,3,4} = \frac{I_{C5}}{2}$$

Dynamic regime

$$A_{dd} = \frac{v_o}{v_1 - v_2} = \frac{i_o R_L}{v_1 - v_2} = \frac{\beta_7 i_{o1} R_L}{v_1 - v_2} \left. \begin{array}{l} i_{o1} = g_{m1}(v_1 - v_2) \end{array} \right\} \Rightarrow A_{dd} = g_{m1} \beta_7 R_L$$

5.3.2. Study of an operational amplifier with three stages

5.3.2. Study of an operational amplifier with three stages



Static regime

$$I_{C6} = \frac{V_{CC} - V_{BE6}}{R_9 + R_{10}} = 1mA$$

$$I_{C5} = I_{C6} \frac{R_{10}}{R_{11}} = 1mA$$

$$I_{C1} = I_{C2} = \frac{I_{C5}}{2} = 0,5mA$$

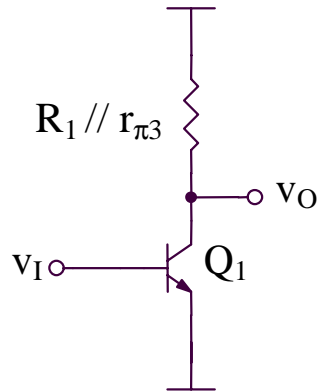
$$I_{C3} = I_{C4} = \frac{V_{CC} - R_2 I_{C2} - V_{BE3}}{2R_6} = 2mA$$

$$I_{C7} = I_{C8} = I_{C9} = \frac{V_{CC} - V_{BE7}}{R_8} = 1mA$$

$$I_{C10} = \frac{2V_{CC} - I_{C4}R_4 - I_{C9}R_7 - V_{BE9} - V_{BE10}}{R_5} \cong 1mA$$

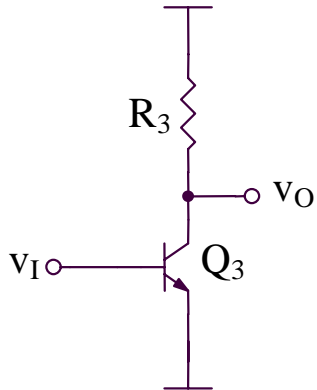
Dynamic regime

Gain of first stage



$$A_{dd I} = -g_{m1}(R_1 // r_{\pi 3} // r_{o1})$$

Gain of second stage



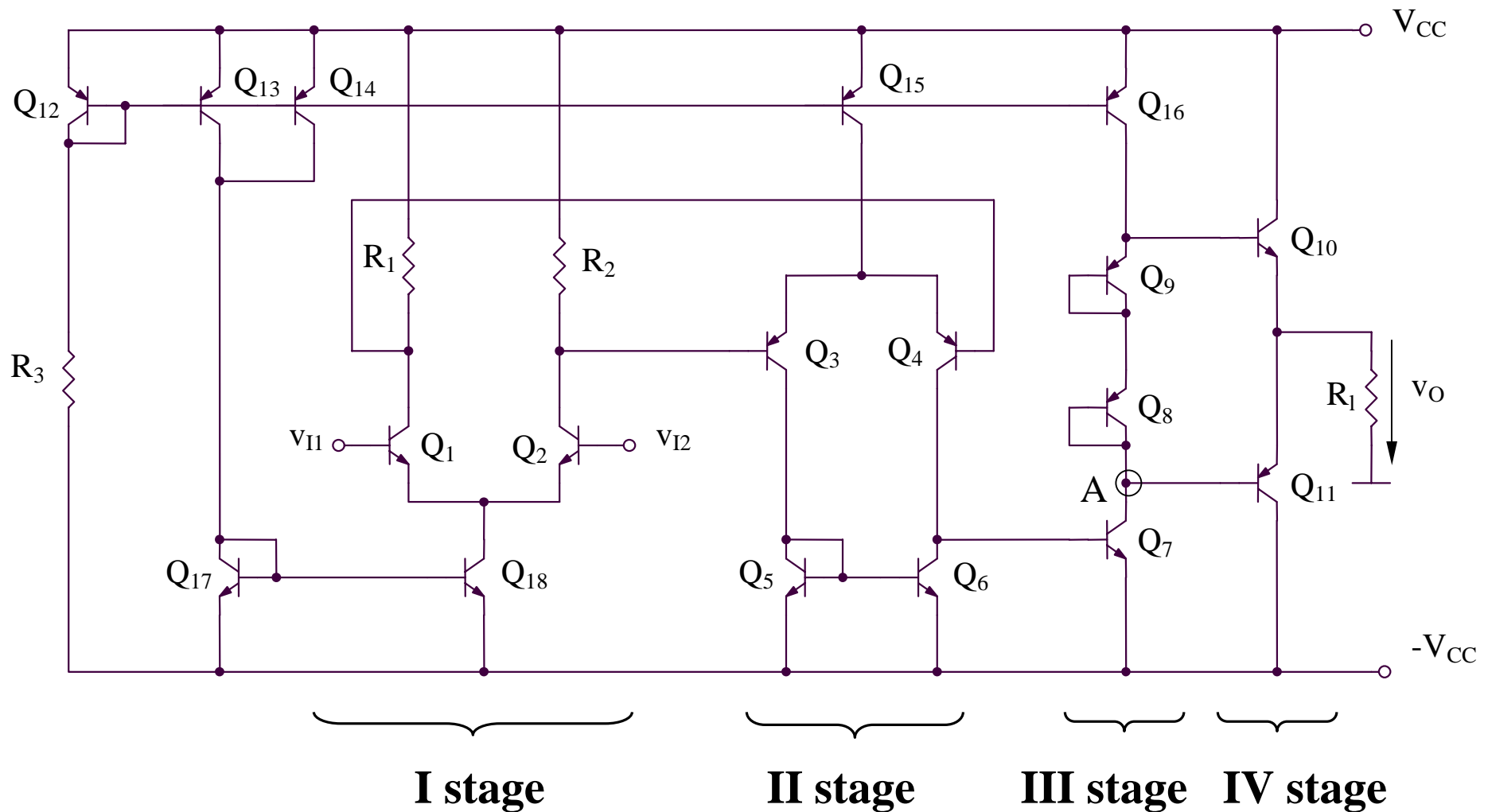
$$A_{dd II} \cong -g_{m3}(R_3 // r_{o3}) \frac{1}{2}$$

Gain of third stage

$$A_{dd III} \cong 1$$

5.3.3. Study of an operational amplifier with four stages

5.3.3. Study of an operational amplifier with four stages



Static regime

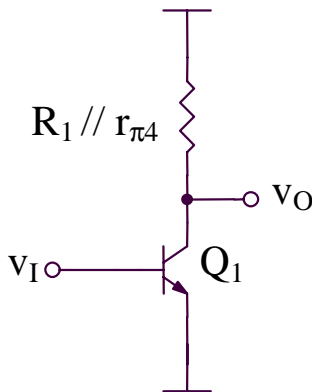
$$I_{C12} = \frac{2V_{CC} - V_{BE}}{R_3} = I_{C13} = I_{C14} = I_{C15} = I_{C16} = I_{C9} = I_{C8} = I_{C7} = I$$

$$I_{C17} = I_{C18} = 2I \quad I_{C3} = I_{C4} = I_{C5} = I_{C6} = I/2 \quad I_{C1} = I_{C2} = I$$

$$|V_{BE8}| + |V_{BE9}| = V_{BE10} + V_{BE11} \Rightarrow 2V_{th} \ln \frac{I}{I_S} = 2V_{th} \ln \frac{I_{C10}}{I_S} \Rightarrow I_{C10} = I_{C11} = I$$

Dynamic regime

Gain of first stage

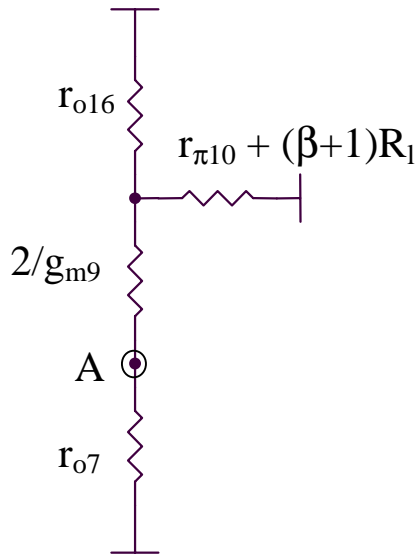


$$A_{dd I} = -g_{m1} (R_1 // r_{\pi 4} // r_{o1})$$

Gain of second stage

$$A_{dd II} = g_{m3} (r_{o6} \parallel r_{o4} \parallel r_{\pi7})$$

Gain of third stage



$$A_{dd III} = -g_{m7} (r_{o7} \parallel r_{o16} \parallel \beta R_l)$$

Gain of fourth stage

$$A_{IV} = \frac{\beta R_l}{r_{\pi10} + \beta R_l} \cong 1$$

Differential mode input resistance

$$R_{id} = 2r_{\pi 1}$$

Maximal common-mode input range

$$V_{IC}^{\min} = V_{BE1} + V_{CE18sat}$$

$$V_{IC}^{\max} = V_{CC} - R_1 I_{C1} - V_{CE1sat} + V_{BE1}$$

Maximal output voltage range

$$V_O^{\max} = \min(V_{CC} - /V_{CE16sat} / - V_{BE10}; I_{C16} \beta R_l)$$

$$V_O^{\min} = -V_{CC} + V_{CE7sat} + /V_{BE11} /$$

5.4. Study of CMOS operational amplifier structures

5.4.1. Study of an operational amplifier with two stages (1)

$$I_{D5} = I_{D6} = I_{D7} = I_{D8} = I_0$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{D7} / 2 = I_0 / 2$$

Dynamic regime

Gain of the circuit

$$A_{dd} = g_{m1}(r_{ds2} // r_{ds4})g_{m5}(r_{ds5} // r_{ds6})$$

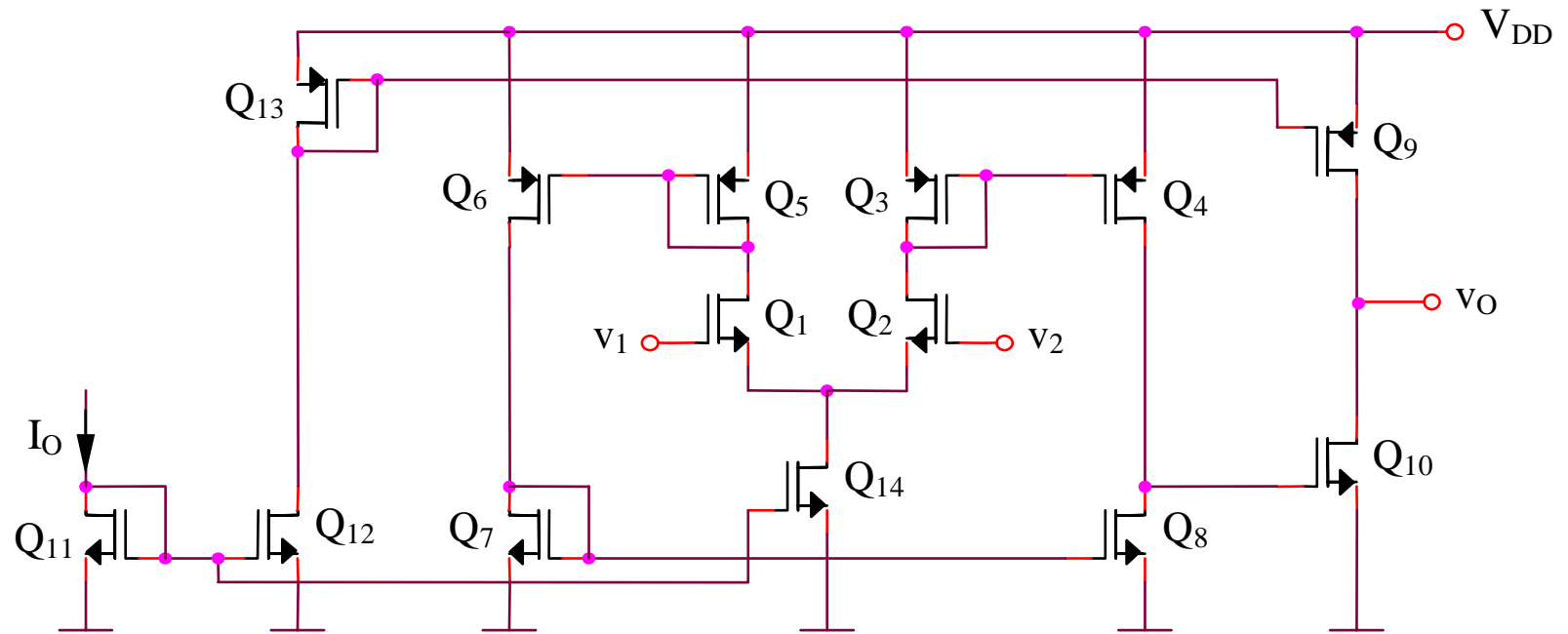
Maximum range of the common-mode input voltage

$$V_{IC}^{\max} = V_{DD} - V_{SG5} - V_{DS2sat} + V_{GS2} = V_{DD} - V_{SG5} + V_T = V_{DD} - \sqrt{\frac{2I_0}{K}}$$

$$V_{IC}^{\min} = V_{DS7sat} + V_{GS1} = V_{GS7} + V_{GS1} - V_T = V_T + (\sqrt{2} + 1)\sqrt{\frac{I_0}{K}}$$

5.4.2. Study of an operational amplifier with two stages (2)

5.4.2. Study of an operational amplifier with two stages (2)



Static regime

$$I_{D1} = \dots = I_{D8} = \frac{I_O}{2}$$

$$I_{D9} = \dots = I_{D14} = I_O$$

Dynamic regime

Gain of the circuit

$$A_{dd} = g_{m1}(r_{ds4} \parallel r_{ds8})g_{m10}(r_{ds10} \parallel r_{ds9}) = \frac{1}{4}g_{m1}g_{m10}r_{ds4}r_{ds10}$$

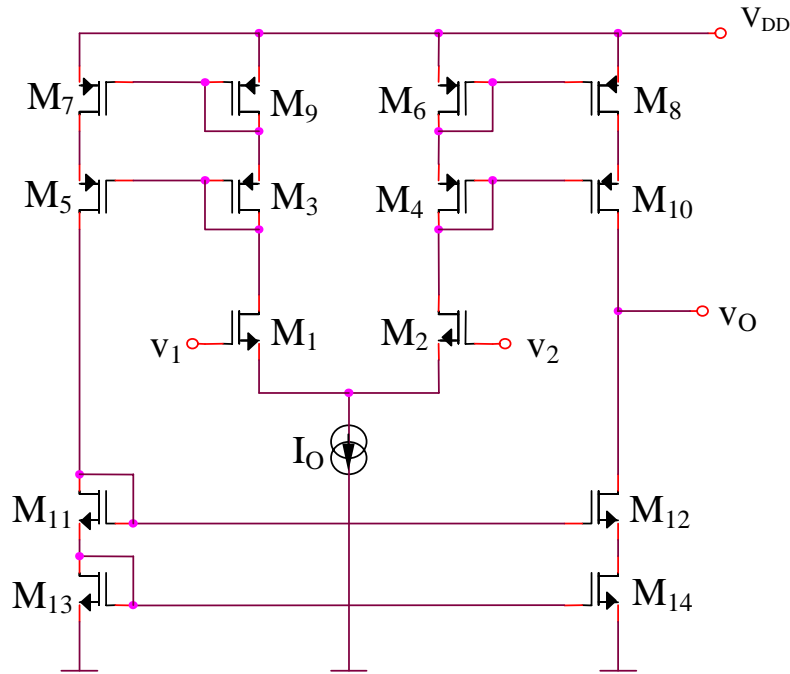
Maximum range of the common-mode input voltage

$$V_{IC}^{\max} = V_{DD} - V_{SG3} - V_{DS2sat} + V_{GS2} = V_{DD} - \sqrt{\frac{I_O}{K}}$$

$$V_{IC}^{\min} = V_{GS2} + V_{DS14sat} = (\sqrt{2} + 1)\sqrt{\frac{I_O}{K}} + V_T$$

5.4.3. Study of a cascode operational amplifier with one stage (1)

5.4.3. Study of a cascode operational amplifier with one stage (1)



Static regime

$$I_{D1} = I_{D2} = \dots = I_{D14} = I_O / 2$$

Dynamic regime

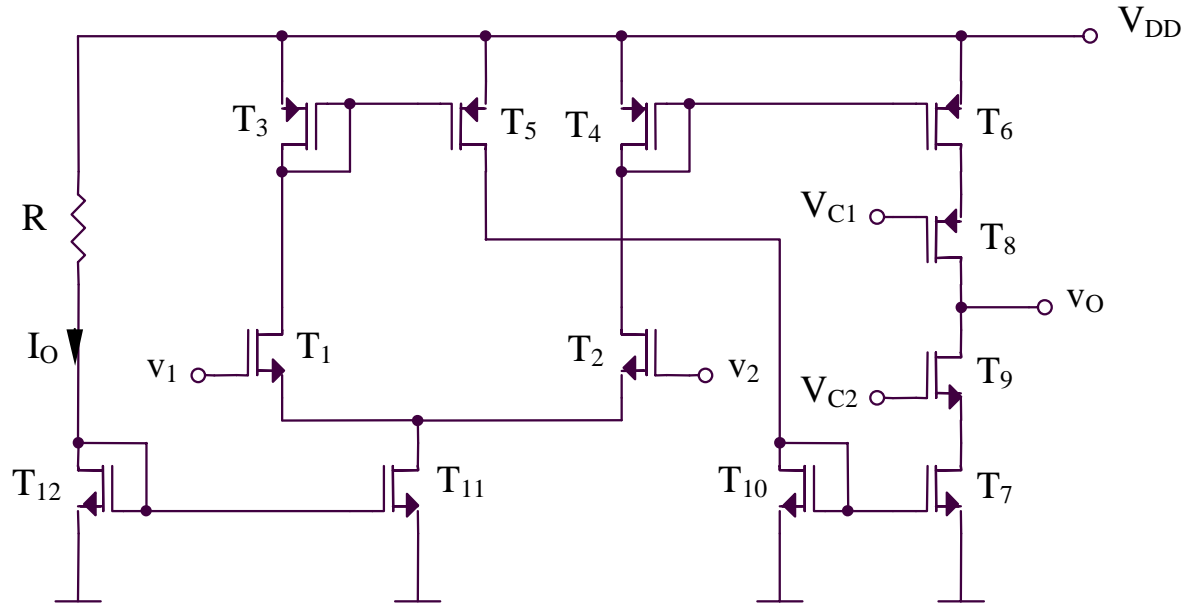
$$R_O = g_{m10} r_{ds10}^2 // g_{m12} r_{ds12}^2 = \frac{1}{2} g_{m10} r_{ds10}^2$$

$$v_O = g_{m1} (v_2 - v_1) R_O = \frac{1}{2} g_{m1} g_{m10} r_{ds10}^2 (v_2 - v_1)$$

$$a = \frac{1}{2} g_{m1} g_{m10} r_{ds10}^2$$

5.4.4. Study of a cascode operational amplifier with one stage (2)

5.4.4. Study of a cascode operational amplifier with one stage (2)



Static regime

$$I_{D1} = I_{D2} = \dots = I_{D10} = I_{D11} / 2 = I_{D12} / 2 = I_O / 2$$

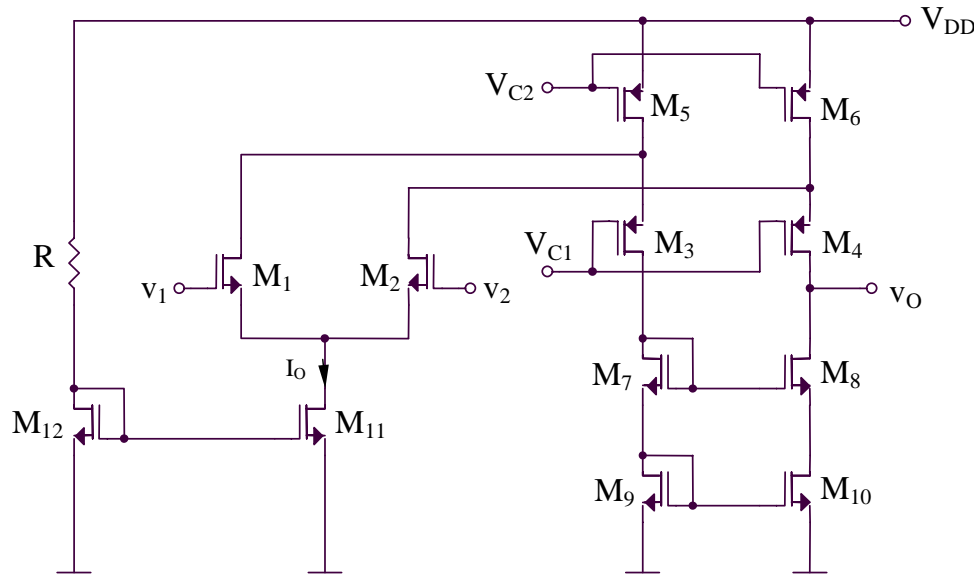
Dynamic regime

$$a = g_{m1} R_O$$

$$R_O = g_{m9} r_{ds9} r_{ds7} // g_{m8} r_{ds8} r_{ds6} = \frac{1}{2} g_{m9} r_{ds9}^2$$

5.4.5. Folded cascod operational amplifier (1)

5.4.5. Folded cascod operational amplifier (1)



Static regime

$$I_{D1} = I_{D2} = I_O / 2$$

$$I_{D11} = I_{D12} = I_O$$

$$I_{D5} = I_{D6} = \frac{K}{2} (V_{DD} - V_{C2} - V_T)^2$$

$$I_{D3} = I_{D4} = I_{D7} = \dots$$

$$\dots = I_{D10} = I_{D6} - I_{D2}$$

Dynamic regime

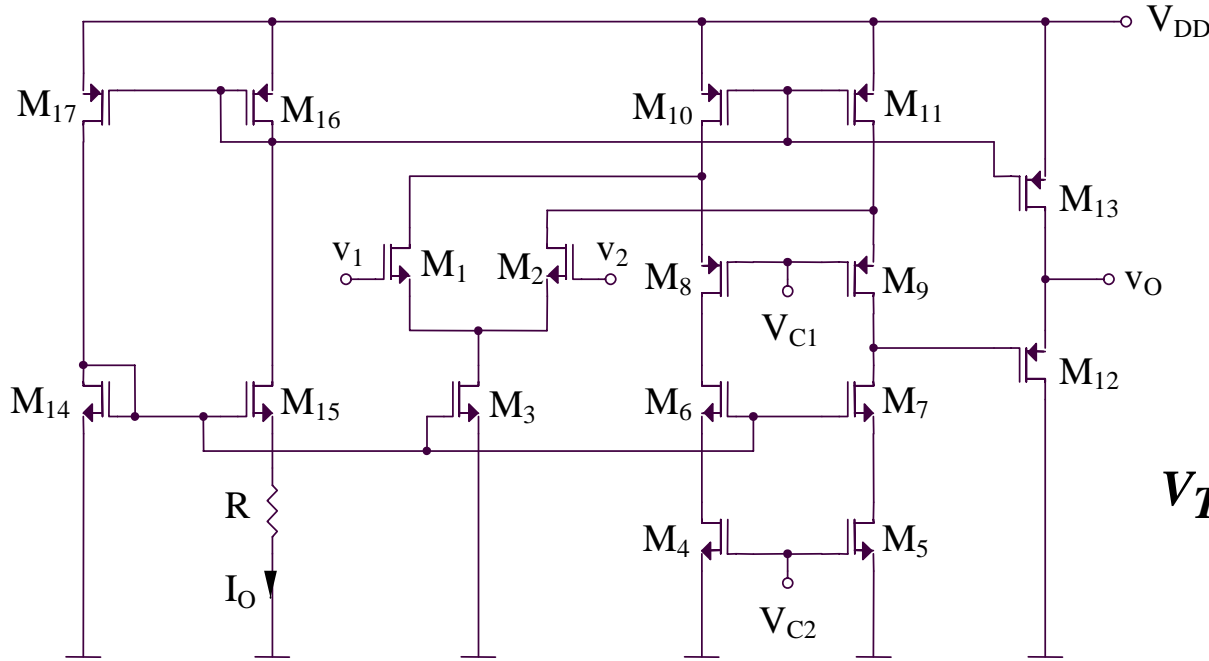
$$i_{D4} - i_{D8} = i_{D4} - i_{D3} = (I_{D6} - i_{D2}) - (I_{D5} - i_{D1}) = i_{D1} - i_{D2}$$

$$a = g_{m1} R_O$$

$$R_O = r_{ds8} g_{m8} r_{ds10} // [r_{ds4} g_{m4} (r_{ds6} // r_{ds2})]$$

5.4.6. Folded cascod operational amplifier (2)

5.4.6. Folded cascod operational amplifier (2)



Static regime

$$V_{C2} \rightarrow I_{D4} = I_{D5} = I_O / 2$$

$$K_{15} = 4K_{14} = 4K$$

$$V_{GS14} = V_{GS15} + I_O R$$

$$V_T + \sqrt{\frac{2I_O}{K}} = V_T + \sqrt{\frac{2I_O}{4K}} + I_O R$$

$$I_O = \frac{1}{2KR^2}$$

$$I_{D1} = I_{D2} = I_{D4} = \dots = I_{D9} = \frac{I_O}{2}$$

$$I_{D3} = I_O = I_{D10} = \dots = I_{D17}$$

Dynamic regime

$$i_{D9} - i_{D7} = i_{D9} - i_{D8} = (I_{D11} - i_{D2}) - (I_{D10} - i_{D1}) = i_{D1} - i_{D2}$$

$$a_1 = g_{m1} R_O$$

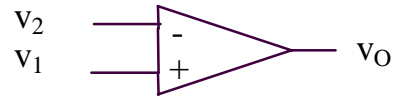
$$a_1 = g_{m1} \{ r_{ds7} g_{m7} r_{ds5} // [r_{ds9} g_{m9} (r_{ds11} // r_{ds2})] \}$$

$$a_2 \cong 1$$

5.5. Comparators

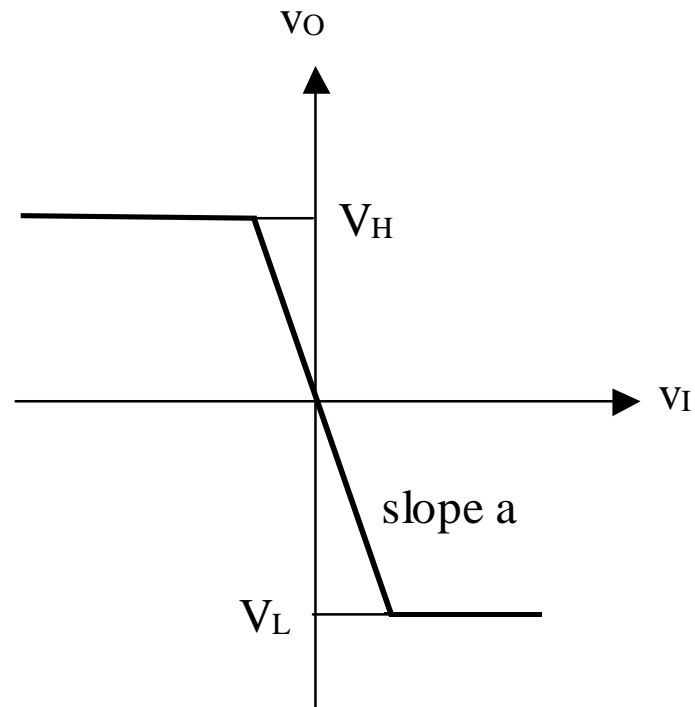
5.5.1. Simple comparator

5.5.1. Simple comparator



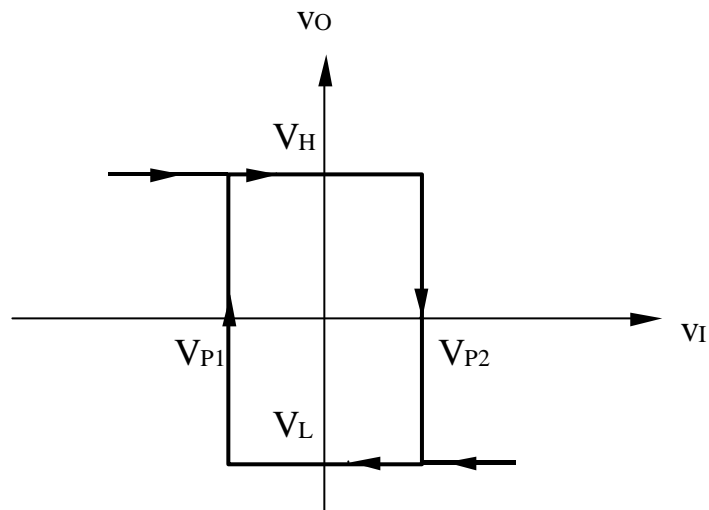
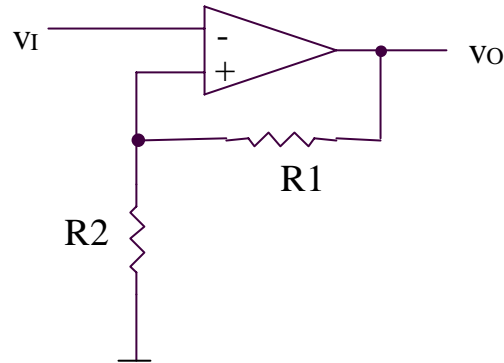
$$v_1 > v_2 \Rightarrow v_O = V_H$$

$$v_1 < v_2 \Rightarrow v_O = V_L$$



5.5.2. Comparator with hysteresis

5.5.2. Comparator with hysteresis



$$V_{P1} = V_H \frac{R_2}{R_1 + R_2}$$

$$V_{P2} = V_L \frac{R_2}{R_1 + R_2}$$