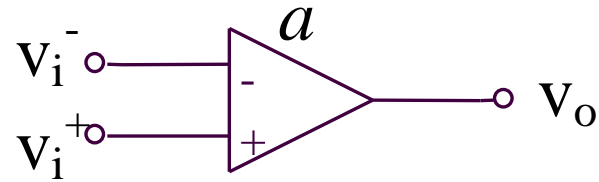


Capitolul 6
Amplificatoare operationale.
Structuri interne

6.1. Introdúcere

6.1. Introducere



$$v_o = a(v_i^+ - v_i^-)$$

Un AO prezinta:

- intrare diferentiala si iesire simpla
- capacitatea de a rejecta semnalele parazite

6.1. Introducere

AO ideal

- impedanta de intrare infinita
- impedanta de iesire nula
- amplificare in tensiune infinita
- banda de frecventa infinita (raspuns instantaneu) – slew rate infinit
- tensiune de iesire nula pentru tensiune de intrare nula
- diferenta de potential nula intre cele doua intrari
- curenti nuli de intrare

AO real:

- impedanta de intrare foarte mare
- impedanta de iesire mica
- amplificare in tensiune foarte mare

Parametri (AO bipolar)

- amplificare in tensiune
- tensiune de intrare de offset
- curent de intrare de offset
- curenti de polarizare a intrarilor
- impedanta de intrare
- domeniul tensiunii de intrare de mod comun
- excursia tensiunii de iesire
- factor de rejectie a tensiunii de mod comun (common-mode rejection ratio)
- factor de rejectie a tensiunii de alimentare
- impedanta de iesire
- frecventa de amplificare unitara (frecventa pentru care amplificarea in bucla deschisa este unitara)
- slew-rate (panta maxima a raspunsului tranzitoriu pentru un semnal mare de intrare de tip treapta)

Ordin de marime

$$a_{v0} > 10^5$$

$$V_{IO} = 2\text{mV}$$

$$I_{IO} = 5\text{nA}$$

$$I_I = 80\text{ nA}$$

$$R_i = 2\text{M}\Omega$$

$$V_{CC} - 2\text{V}$$

$$V_{CC} - (2 \dots 3V_{BE}) - V_{CEsat}$$

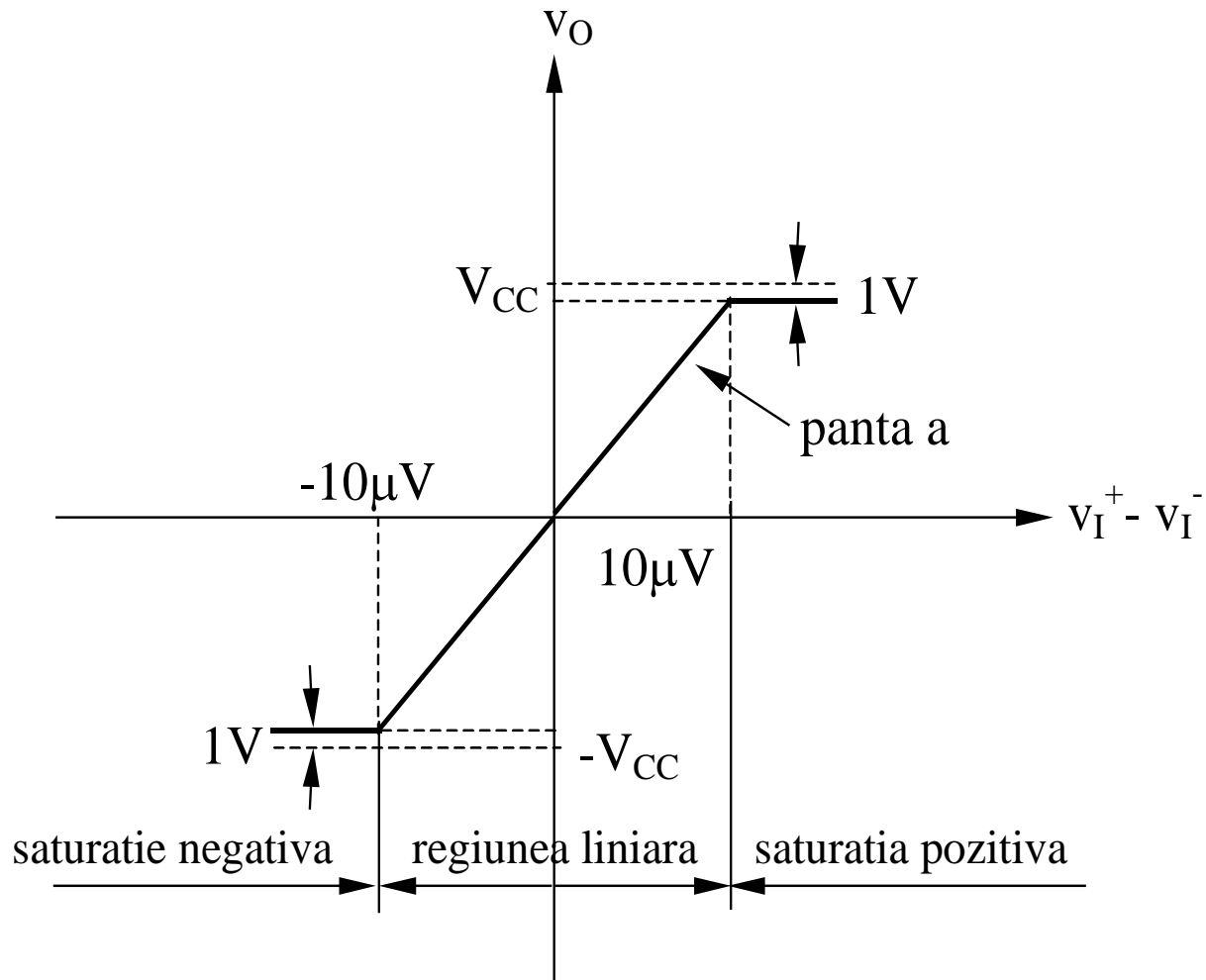
$$\text{CMRR} = 80\text{dB}$$

$$\text{PSRR} = 80\text{-}120\text{dB}$$

$$R_O = 75\Omega$$

$$f_T = 1\text{MHz}$$

$$\text{SR} = (0,2\text{-}1)\text{V}/\mu\text{s}$$



In regiunea liniara, $v_O = a(v_I^+ - v_I^-)$
 In saturatia negativa, $v_O \cong -(V_{CC} - IV)$
 In saturatia pozitiva, $v_O \cong V_{CC} - IV$

6.2. Deviatii de la idealitate si limitari ale AO real

6.2. Deviati de la idealitate si limitari ale AO real

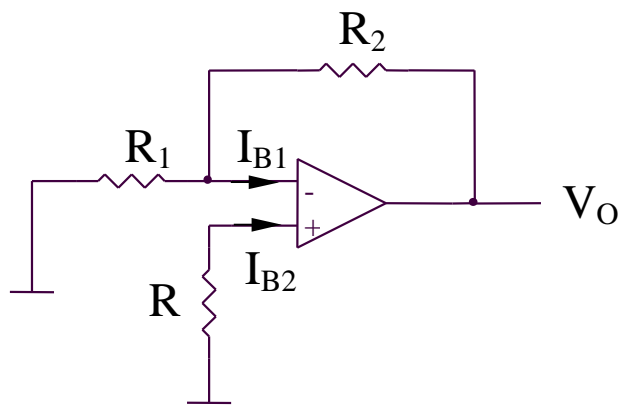
6.2.1. Curentul de polarizare a intrarilor (I_B)

Reprezinta media aritmetica a curentilor de intrare intr-un amplificator diferential.

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

Valori tipice:

- 10-100 nA – pentru etaje de intrare realizate in tehnologia bipolară
- < 0,001pA – pentru etaje de intrare realizate in tehnologia MOS



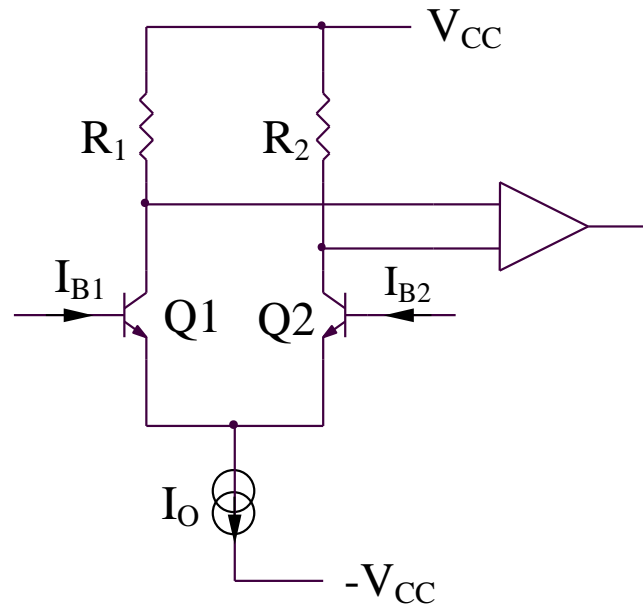
$$V_O = -I_{B2}R \left(1 + \frac{R_2}{R_1} \right) + I_{B1}R_2$$

$$\text{Daca: } I_{B1} = I_{B2} \text{ si } R = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$\text{se obtine: } V_O = 0$$

6.2. Deviatii de la idealitate si limitari ale AO real

6.2.2. Curentul de offset (decalaj) de intrare (I_{OS})

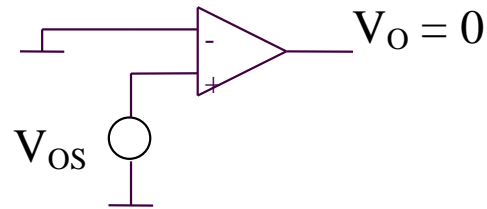


$$I_{OS} = I_{B1} - I_{B2}$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.3. Tensiunea de offset (decalaj) de intrare (V_{OS})

Reprezinta tensiunea care trebuie aplicata intre intrarile unui AO real pentru a obtine o tensiune de iesire nula.



Valori tipice pentru V_{OS} :

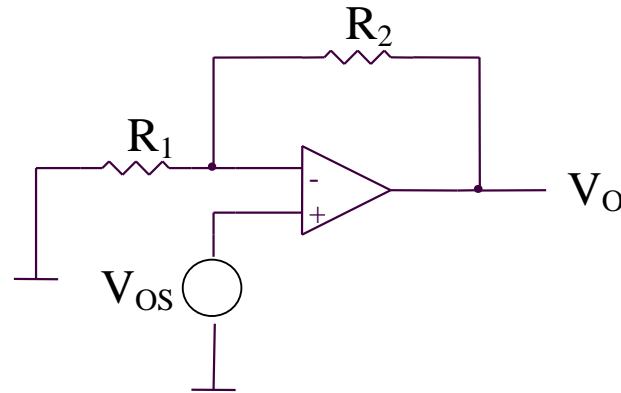
- pentru AO bipolare: $0,1 \div 2$ mV
- pentru AO CMOS: $1 \div 20$ mV

V_{OS} prezinta o importanta dependenta de temperatura.

6.2. Deviatii de la idealitate si limitari ale AO real

6.2.3. Tensiunea de offset (decalaj) de intrare (V_{OS})

Erori introduse de tensiunea de offset in functionarea amplificatorului neinversor

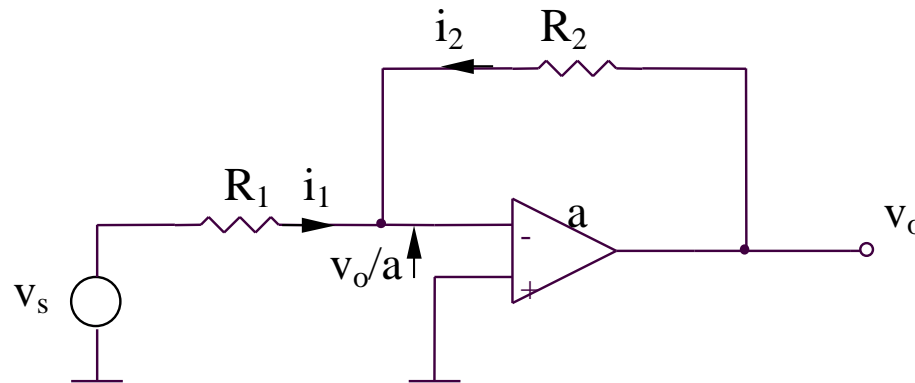


$$V_O = \left(1 + \frac{R_2}{R_1} \right) V_{OS}$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.4. Valoarea finita a amplificarii in bucla deschisa (a)

Amplificatorul inversor



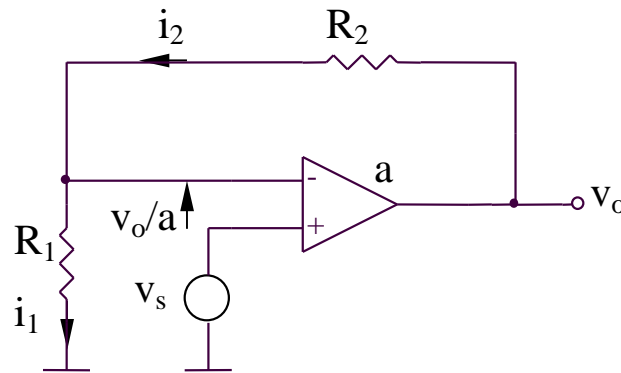
$$\frac{v_s + \frac{v_o}{a}}{R_1} + \frac{v_o + \frac{v_o}{a}}{R_2} = 0 \Rightarrow$$

$$\Rightarrow A = \frac{v_o}{v_s} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{1}{a} \frac{R_1 + R_2}{R_1}} \xrightarrow{a \rightarrow \infty} -\frac{R_2}{R_1}$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.4. Valoarea finita a amplificarii in bucla deschisa (a)

Amplificatorul neinversor



$$\frac{v_s - \frac{v_o}{a}}{R_1} = \frac{v_o - \left(v_s - \frac{v_o}{a} \right)}{R_2}$$

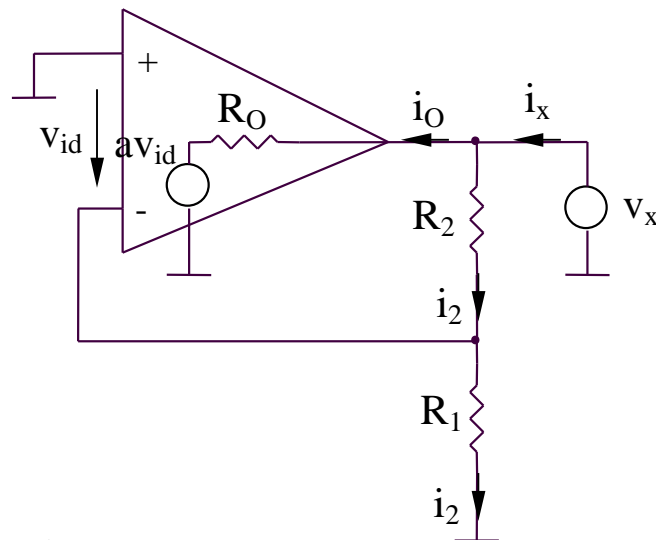
$$A = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1} \frac{1}{1 + \frac{R_1 + R_2}{aR_1}} \xrightarrow{a \rightarrow \infty} 1 + \frac{R_2}{R_1}$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.5. Valoarea nenula a rezistentei de iesire a AO (R_O)

Valori tipice:

- pentru AO bipolare: $40 \Omega - 100 \Omega$
- pentru AO CMOS: rezistente de iesire mult mai mari (OTA), deoarece AO nu comanda uzual sarcini rezistive



$$R_{OUT} = \frac{v_x}{i_x}$$

$$i_x = i_o + i_2 = \frac{v_x - av_{id}}{R_O} + \frac{v_x}{R_1 + R_2}$$

$$v_{id} = -v_x \frac{R_1}{R_1 + R_2} = -fv_x$$

Rezulta:

$$\frac{1}{R_{OUT}} = \frac{i_x}{v_x} = \frac{1+af}{R_O} + \frac{1}{R_1 + R_2}$$

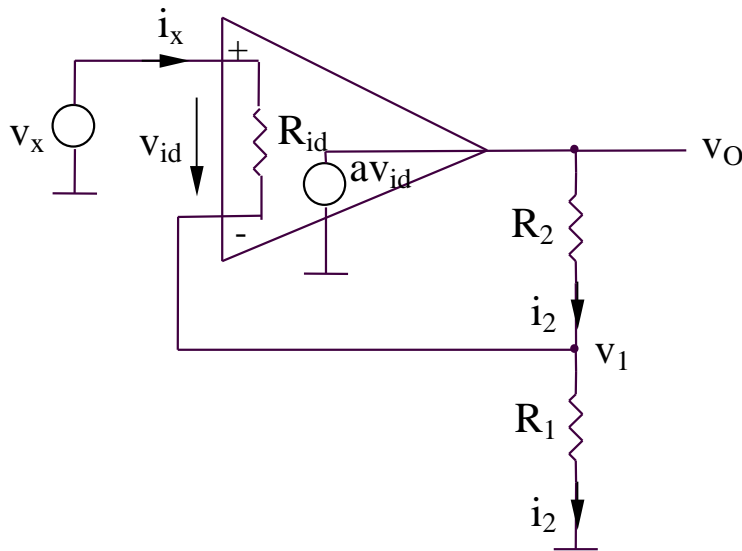
$$R_{OUT} \cong \frac{R_O}{1+af} \text{ pentru } \frac{R_O}{1+af} \ll R_1 + R_2$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.6. Valoarea nenula a rezistentei de intrare a AO (R_i)

Valori tipice:

- pentru AO bipolare: 100 k Ω - 1 M Ω
- pentru AO CMOS: rezistenta de intrare infinita



$$R_{IN} = \frac{v_x}{i_x}$$

$$i_x = \frac{v_x - v_1}{R_{id}}$$

$$v_1 = v_o \frac{R_1}{R_1 + R_2} = f(av_{id}) = af(v_x - v_1)$$

$$\Rightarrow v_1 = \frac{af}{1 + af} v_x$$

$$R_{IN} = \frac{v_x}{i_x} = R_{id}(1 + af)$$

6.2. Deviati de la idealitate si limitari ale AO real

6.2.7. Raportul de rejectie a modului comun (CMRR)

Caracterizeaza capacitatea amplificatorului operational de a amplifica semnalele de mod diferential si de a rejecta semnalele de mod comun.

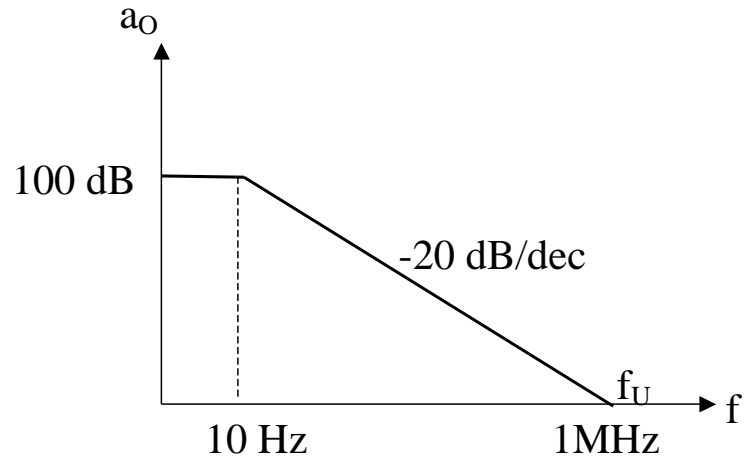
6.2.8. Raportul de rejectie a tensiunii de alimentare (PSRR)

Caracterizeaza capacitatea amplificatorului operational de a rejecta variatiile tensiunii de alimentare.

6.2. Deviati de la idealitate si limitari ale AO real

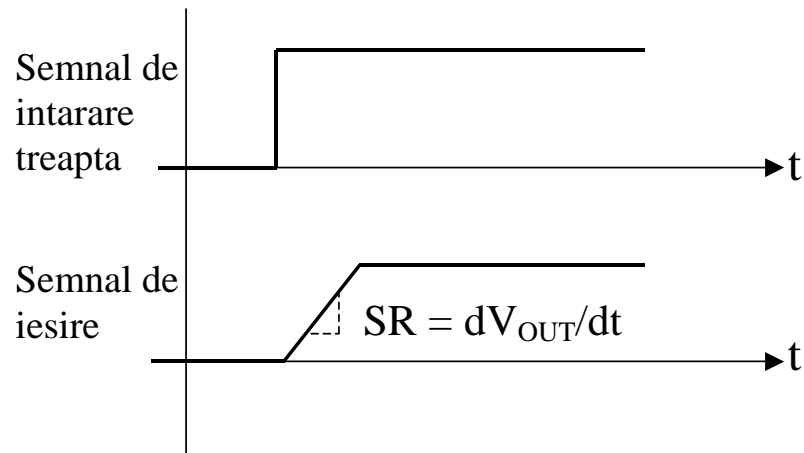
6.2.9. Raspunsul in frecventa

Amplificarea in tensiune a unui AO scade la cresterea frecventei, ca o consecinta a capacitatilor interne ale dispozitivelor.



f_U – in domeniul 1MHz – 100MHz

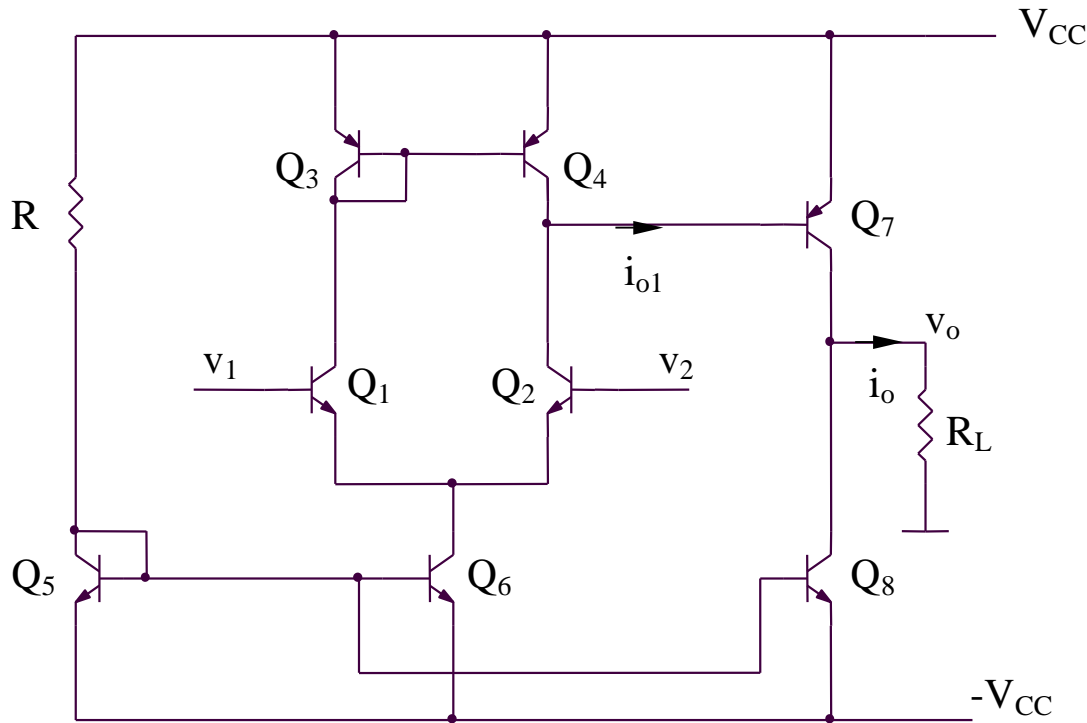
Exista o limitare a raspunsului in frecventa din cauza vitezei de crestere (slew-rate - SR), in conditii de functionare la semnal mare.



6.3. Structuri interne de amplificatoare operationale

Amplificatoare operationale in tehnologie bipolară

6.3.1. Amplificator operational cu 2 etaje de amplificare



Regim static

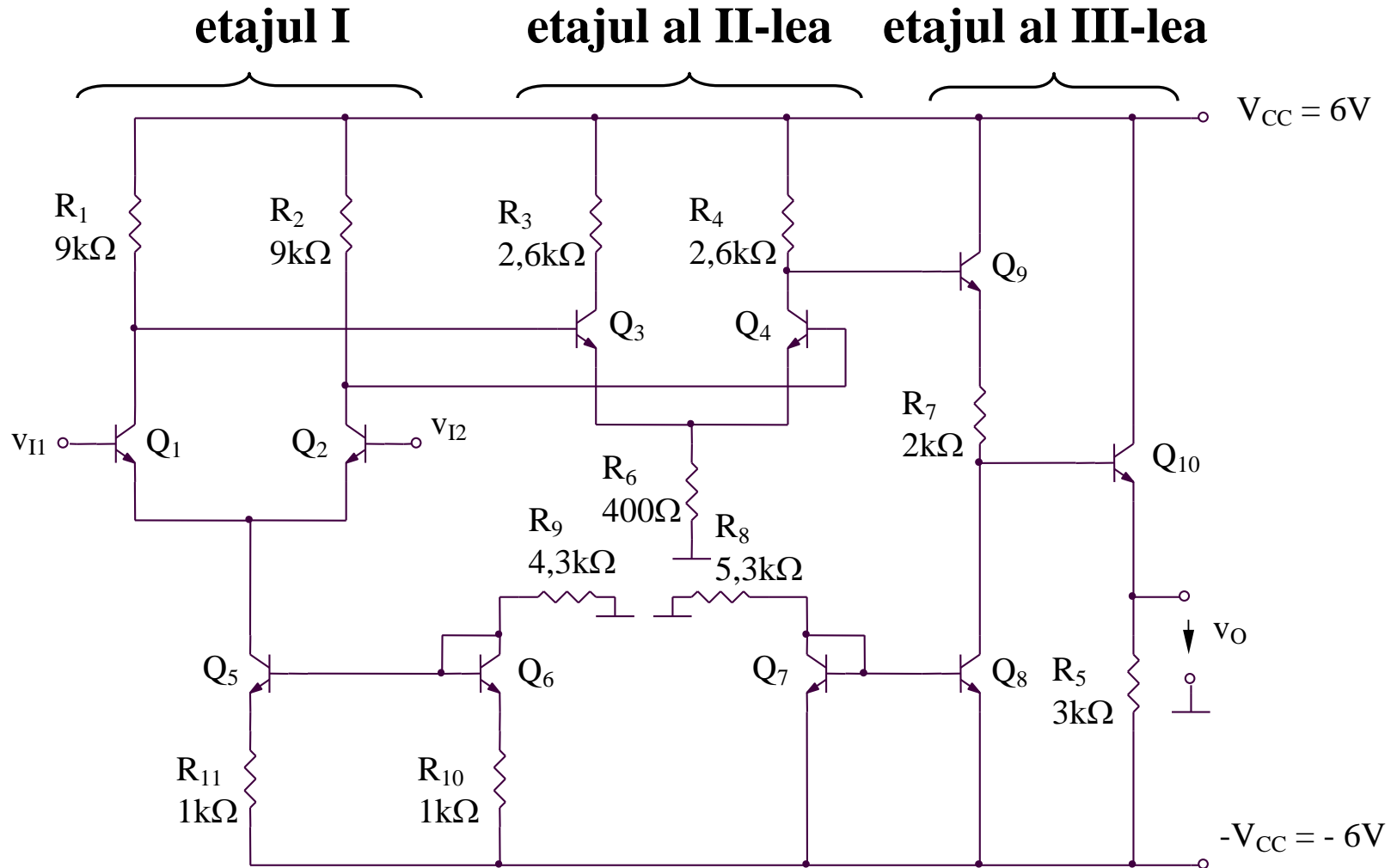
$$I_{C5,6,7,8} = \frac{2V_{CC} - V_{BE}}{R}$$

$$I_{C1,2,3,4} = \frac{I_{C5}}{2}$$

Regim dinamic

$$A_{dd} = \frac{v_o}{v_1 - v_2} = \frac{i_o R_L}{v_1 - v_2} = \frac{\beta_7 i_{o1} R_L}{v_1 - v_2} \left. \vphantom{A_{dd}} \right\} \Rightarrow A_{dd} = g_{m1} \beta_7 R_L$$

6.3.2. Amplificator operational cu 3 etaje de amplificare



Regim static

$$I_{C6} = \frac{V_{CC} - V_{BE6}}{R_9 + R_{10}} = 1mA$$

$$I_{C5} = I_{C6} \frac{R_{10}}{R_{11}} = 1mA$$

$$I_{C1} = I_{C2} = \frac{I_{C5}}{2} = 0,5mA$$

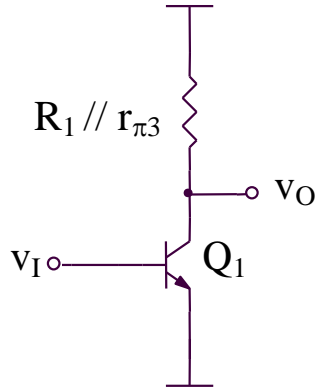
$$I_{C3} = I_{C4} = \frac{V_{CC} - R_2 I_{C2} - V_{BE3}}{2R_6} = 2mA$$

$$I_{C7} = I_{C8} = I_{C9} = \frac{V_{CC} - V_{BE7}}{R_8} = 1mA$$

$$I_{C10} = \frac{2V_{CC} - I_{C4}R_4 - I_{C9}R_7 - V_{BE9} - V_{BE10}}{R_5} \cong 1mA$$

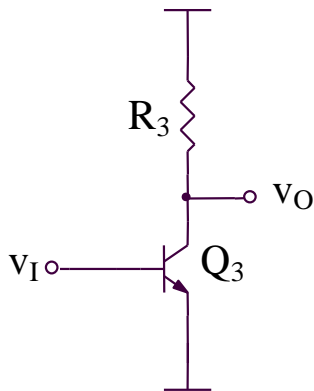
Regim dinamic (analiza de mod differential)

Semicircuitul de mod differential (etajul I)



$$A_{dd I} = -g_{m1}(R_1 // r_{\pi 3} // r_{o1})$$

Semicircuitul de mod differential (etajul al II-lea)



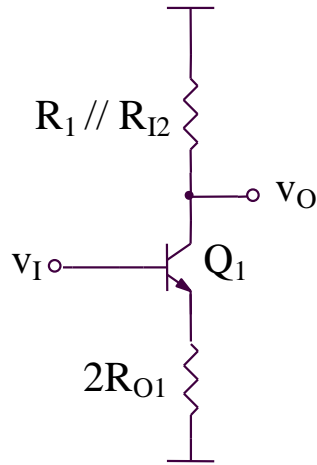
$$A_{dd II} \cong -g_{m3}(R_3 // r_{o3}) \frac{1}{2}$$

Etajul al III-lea

$$A_{dd III} \cong 1$$

Regim dinamic (analiza de mod comun)

Semicircuitul de mod comun (etajul I)

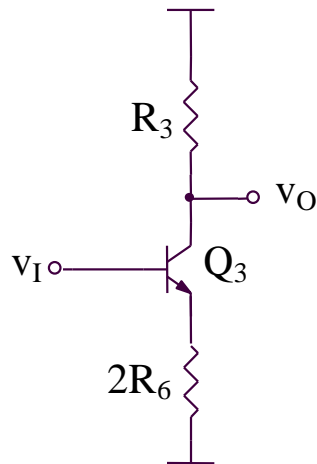


$$A_{cc I} = -\frac{\beta(R_1 // R_{I2})}{r_{\pi 1} + (\beta + 1)2R_{O1}}$$

$$R_{I2} = r_{\pi 3} + (\beta + 1)2R_6$$

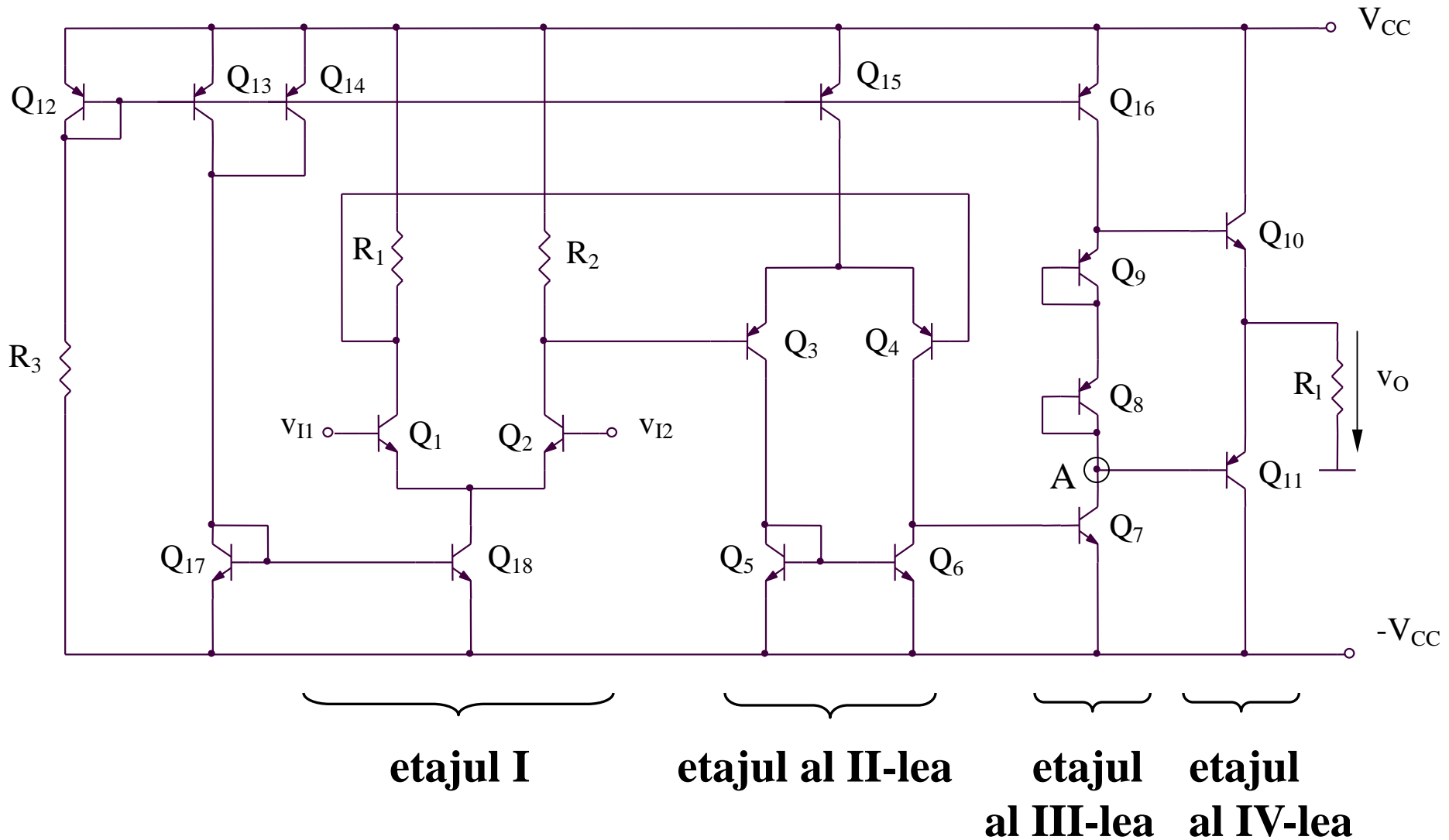
$$R_{O1} = r_{o5} \left(1 + \frac{\beta R_{11}}{r_{\pi 5} + R_{11} + (1/g_{m6} + R_{10}) // R_9} \right)$$

Semicircuitul de mod comun (etajul al II-lea)



$$A_{cc II} = -\frac{\beta R_3}{r_{\pi 3} + (\beta + 1)2R_6}$$

6.3.3. Amplificator operational cu 4 etaje de amplificare



Regim static

$$I_{C12} = \frac{2V_{CC} - V_{BE}}{R_3} = I_{C13} = I_{C14} = I_{C15} = I_{C16} = I_{C9} = I_{C8} = I_{C7} = I$$

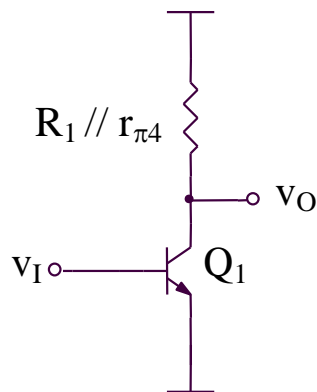
$$I_{C17} = I_{C18} = 2I \quad I_{C3} = I_{C4} = I_{C5} = I_{C6} = I/2 \quad I_{C1} = I_{C2} = I$$

$$/V_{BE8} / + /V_{BE9} / = V_{BE10} + V_{BE11} \Rightarrow 2V_{th} \ln \frac{I}{I_{S8}} = 2V_{th} \ln \frac{I_{C10}}{I_{S10}} \Rightarrow$$

$$\Rightarrow I_{C10} = I_{C11} = I \frac{I_{S10}}{I_{S8}} = I \frac{A_{10}}{A_8}$$

Regim dinamic

Amplificarea primului etaj

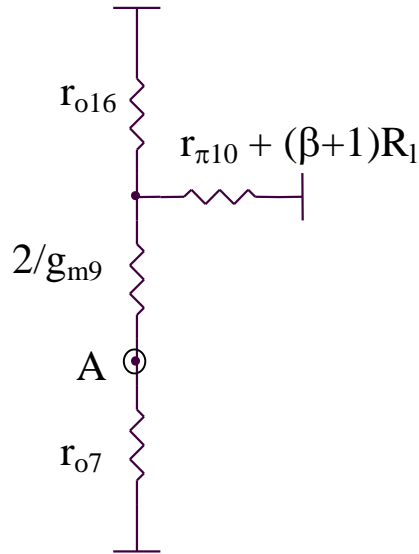


$$A_{dd I} = -g_{m1} (R_1 // r_{\pi 4} // r_{o1})$$

Amplificarea etajului al II-lea

$$A_{dd\ II} = g_{m3} (r_{o6} \parallel r_{o4} \parallel r_{\pi7})$$

Amplificarea etajului al III-lea



$$A_{dd\ III} = -g_{m7} (r_{o7} \parallel r_{o16} \parallel \beta R_1)$$

Amplificarea etajului al IV-lea

$$A_{IV} = \frac{\beta R_1}{r_{\pi10} + \beta R_1} \cong 1$$

Rezistența de intrare de mod diferential

$$R_{id} = 2r_{\pi 1}$$

Domeniul maxim al tensiunii de intrare de mod comun

$$V_{IC}^{min} = -V_{CC} + V_{BE1} + V_{CE18sat}$$

$$V_{IC}^{max} = V_{CC} - R_1 I_{C1} - V_{CE1sat} + V_{BE1}$$

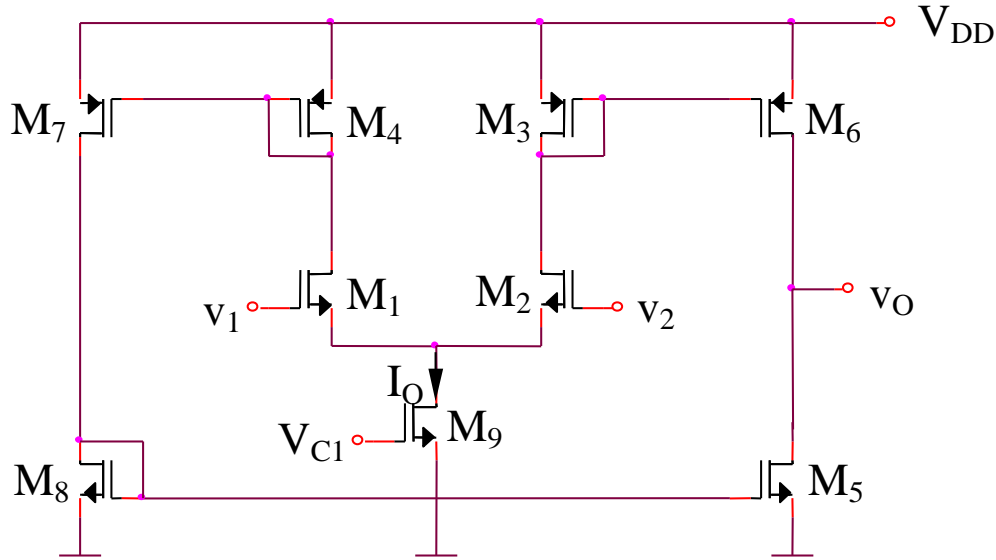
Excursia maxima a tensiunii de iesire

$$V_O^{max} = \min(V_{CC} - /V_{CE16sat} / - V_{BE10}; I_{C16} \beta R_l)$$

$$V_O^{min} = -V_{CC} + V_{CE7sat} + /V_{BE11} /$$

Amplificatoare operationale clasice in tehnologie CMOS

6.3.4. Amplificator operational cu un etaj de amplificare



Regim static

$$I_{D9} = I_O = \frac{K}{2} (V_{C1} - V_T)^2$$

$$I_{D1} = I_{D2} = \dots = I_{D8} = I_O / 2$$

Regim dinamic

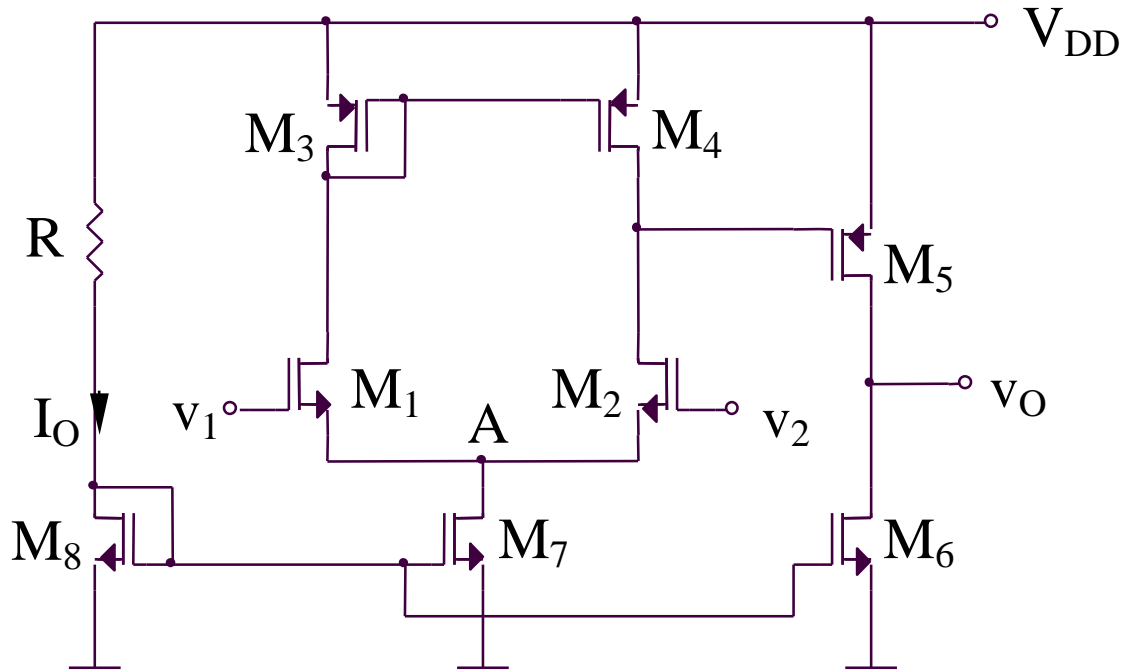
$$i_{D6} - i_{D5} = i_{D3} - i_{D8} = i_{D3} - i_{D4} = i_{D2} - i_{D1} = g_{m1}(v_2 - v_1)$$

$$v_O = (i_{D6} - i_{D5})R_O = g_{m1}(v_2 - v_1)R_O \quad R_O = r_{ds6} // r_{ds5}$$

$$v_O = g_{m1}(v_2 - v_1)R_O = \frac{1}{2} g_{m1} r_{ds5} (v_2 - v_1)$$

$$a = \frac{1}{2} g_{m1} r_{ds5} = \frac{1}{\lambda} \sqrt{\frac{K}{I_O}}$$

6.3.5. Amplificator operational cu 2 etaje de amplificare (1)



Regim static

$$\left. \begin{aligned} RI_O + V_{GS8} &= V_{DD} \\ I_O &= \frac{K}{2} (V_{GS8} - V_T)^2 \end{aligned} \right\} \Rightarrow V_{DD} = V_{GS8} + \frac{RK}{2} (V_{GS8} - V_T)^2 \Rightarrow V_{GS8} \quad (V_{GS8} > V_T)$$

$$\Rightarrow I_O = \frac{K}{2} (V_{GS8} - V_T)^2$$

$$I_{D5} = I_{D6} = I_{D7} = I_{D8} = I_O$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{D7} / 2 = I_O / 2$$

Regim dinamic

Amplificarea circuitului

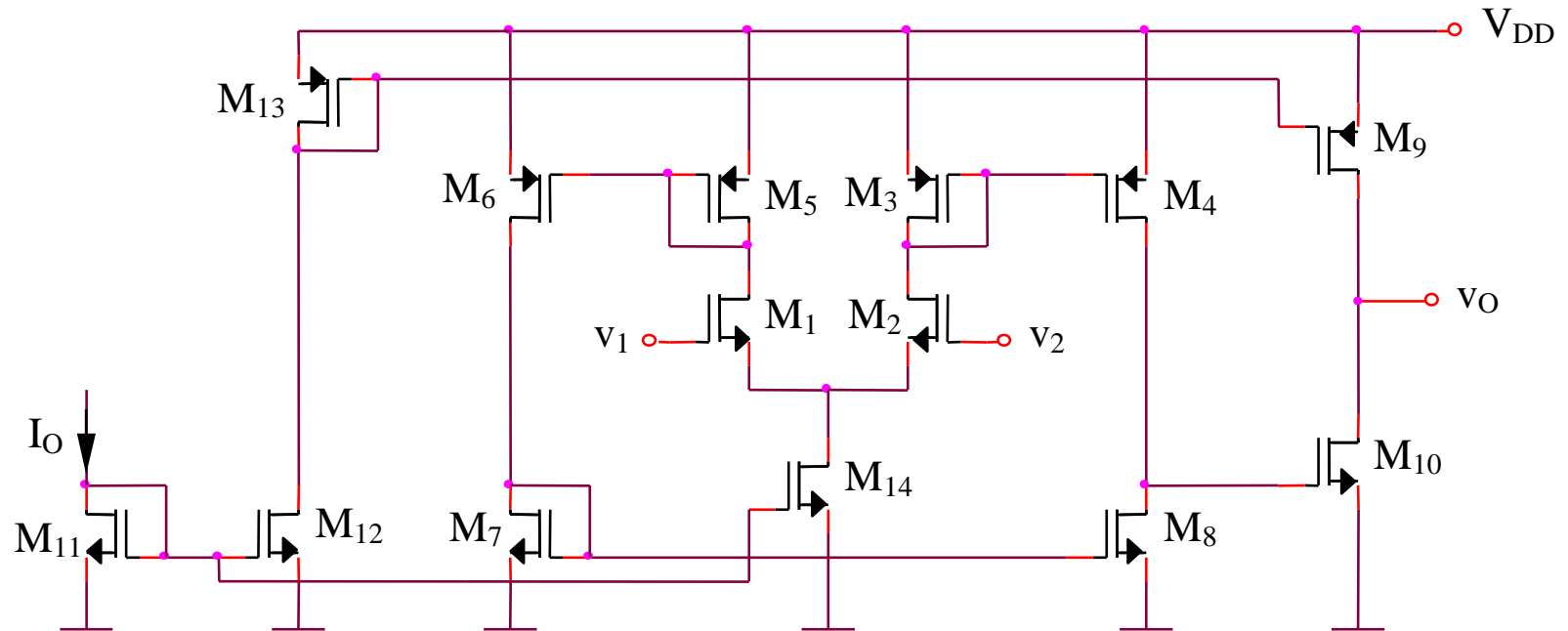
$$a = g_{m1}(r_{ds2} // r_{ds4})g_{m5}(r_{ds5} // r_{ds6})$$

Domeniul maxim al tensiunii de intrare de mod comun

$$V_{IC}^{\max} = V_{DD} - V_{SG5} - V_{DS2sat} + V_{GS2} = V_{DD} - V_{SG5} + V_T = V_{DD} - \sqrt{\frac{2I_O}{K}}$$

$$V_{IC}^{\min} = V_{DS7sat} + V_{GS1} = V_{GS7} + V_{GS1} - V_T = V_T + (\sqrt{2} + 1)\sqrt{\frac{I_O}{K}}$$

6.3.6. Amplificator operational cu 2 etaje de amplificare (2)



Regim static

$$I_{D1} = \dots = I_{D8} = \frac{I_O}{2}$$

$$I_{D9} = \dots = I_{D14} = I_O$$

Regim dinamic

Amplificarea circuitului

$$a = g_{m1}(r_{ds4} // r_{ds8})g_{m10}(r_{ds10} // r_{ds9})$$

Domeniul maxim al tensiunii de intrare de mod comun

$$V_{IC}^{\max} = V_{DD} - V_{SG3} - V_{DS2sat} + V_{GS2} = V_{DD} - \sqrt{\frac{I_O}{K}}$$

$$V_{IC}^{\min} = V_{GS2} + V_{DS14sat} = (\sqrt{2} + 1)\sqrt{\frac{I_O}{K}} + V_T$$

6.3.7. Amplificator operational cu 3 etaje de amplificare

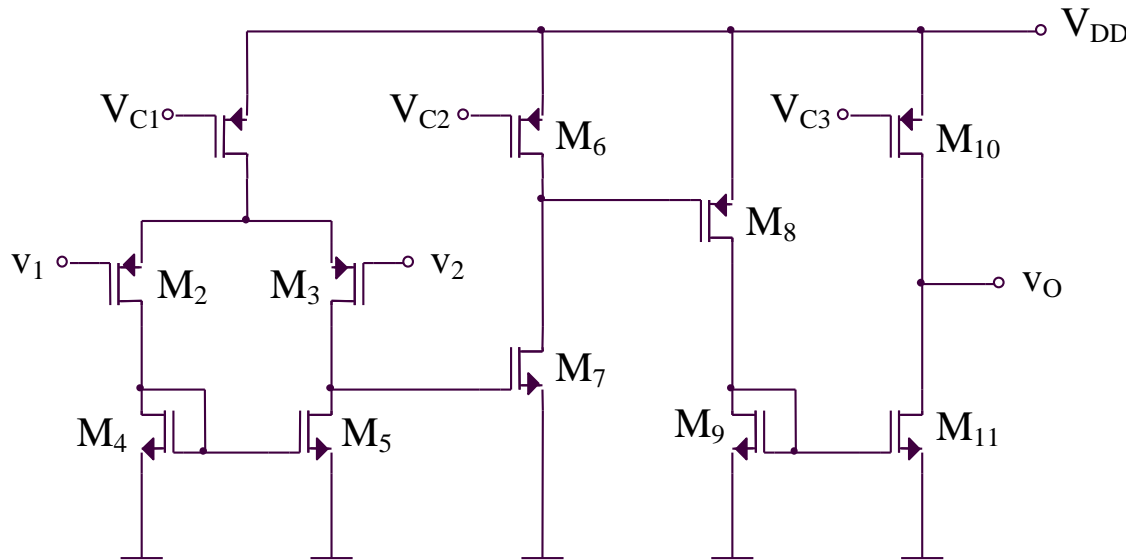
Regim static

$$I_{D1} = \frac{K}{2}(V_{DD} - V_{C1} - V_T)^2$$

$$I_{D6} = I_{D7} = \frac{K}{2}(V_{DD} - V_{C2} - V_T)^2$$

$$I_{D10} = I_{D11} = I_{D8} = I_{D9} = \frac{K}{2}(V_{DD} - V_{C3} - V_T)^2$$

$$I_{D2} = I_{D3} = I_{D4} = I_{D5} = \frac{I_{D1}}{2}$$



Regim dinamic

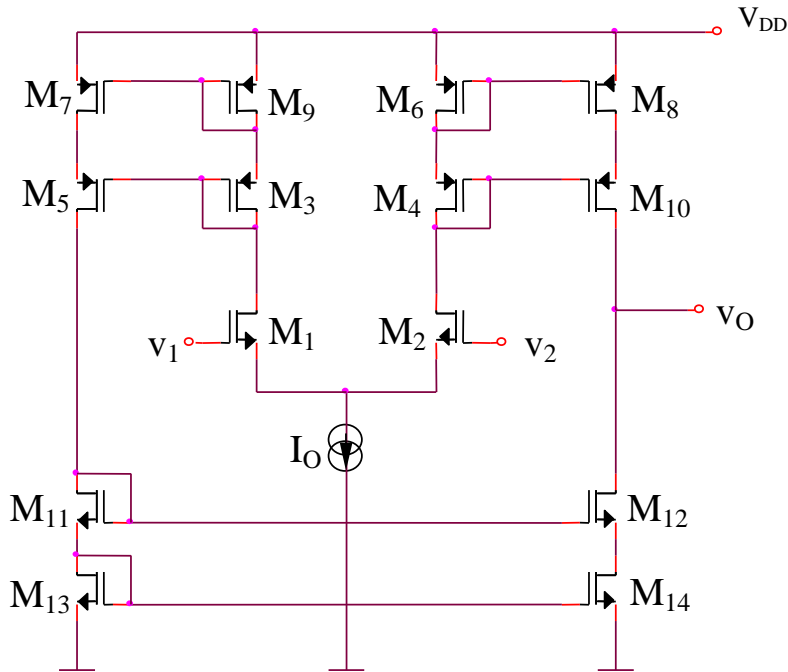
$$A_{dd1} = g_{m2}(r_{ds3} // r_{ds5})$$

$$A_{dd2} = -g_{m7}(r_{ds6} // r_{ds7})$$

$$A_{dd3} = -g_{m8}(r_{ds10} // r_{ds11})$$

Amplificatoare operationale cascod in tehnologie CMOS

6.3.8. Amplificator operational cascod cu un etaj de amplificare (1)



Regim static

$$I_{D1} = I_{D2} = \dots = I_{D14} = I_O / 2$$

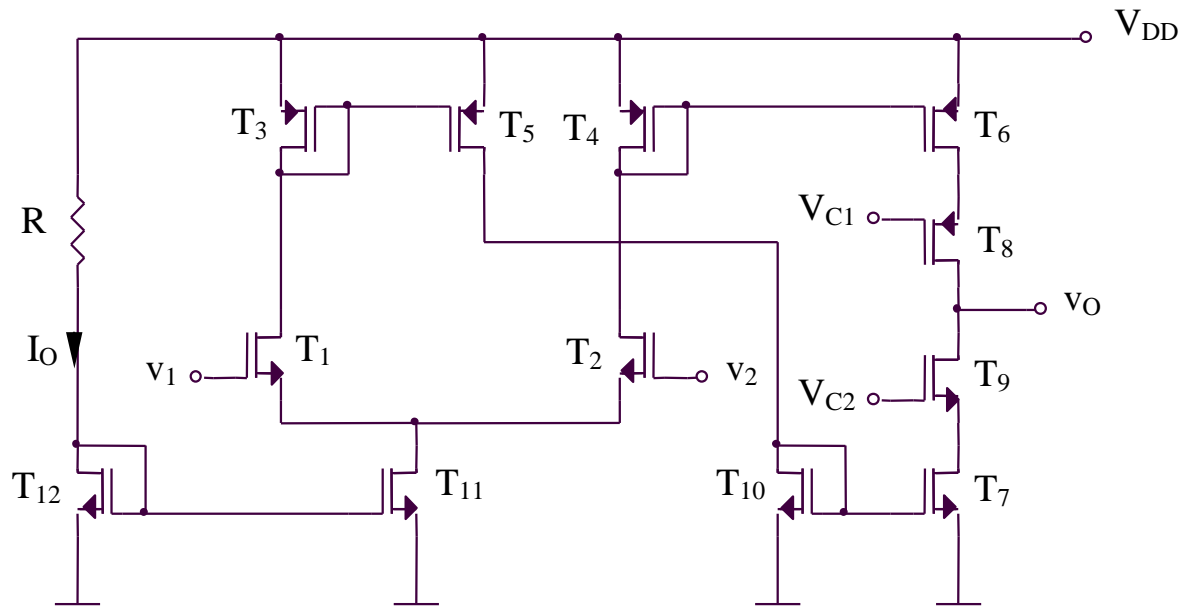
Regim dinamic

$$R_O = g_{m10} r_{ds10}^2 // g_{m12} r_{ds12}^2$$

$$v_O = g_{m1} (v_2 - v_1) R_O = \frac{1}{2} g_{m1} g_{m10} r_{ds10}^2 (v_2 - v_1)$$

$$a = \frac{1}{2} g_{m1} g_{m10} r_{ds10}^2$$

6.3.9. Amplificator operational cascod cu un etaj de amplificare (2)



Regim static

$$I_{D1} = I_{D2} = \dots = I_{D10} = I_{D11} / 2 = I_{D12} / 2 = I_O / 2$$

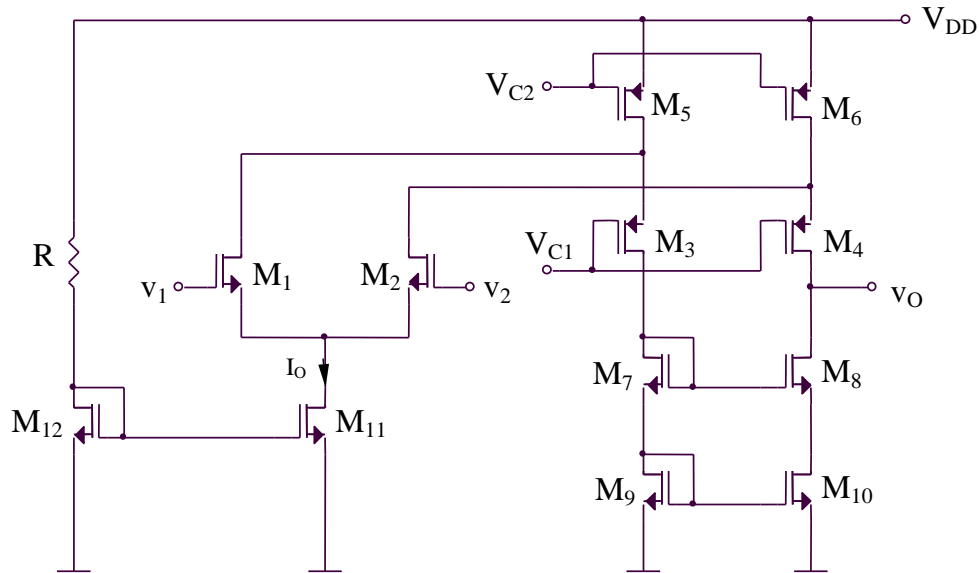
Regim dinamic

$$a = g_{m1} R_O$$

$$R_O = g_{m9} r_{ds9} r_{ds7} // g_{m8} r_{ds8} r_{ds6} = \frac{1}{2} g_{m9} r_{ds9}^2$$

*Amplificatoare operationale cascoda intoarsa (folded cascod)
in tehnologie CMOS*

6.3.10. Amplificator operational cascada intoarsa (1)



Regim static

$$I_{D1} = I_{D2} = I_O / 2$$

$$I_{D11} = I_{D12} = I_O$$

$$I_{D5} = I_{D6} = \frac{K}{2} (V_{DD} - V_{C2} - V_T)^2$$

$$I_{D3} = I_{D4} = I_{D7} = \dots$$

$$\dots = I_{D10} = I_{D6} - I_{D2}$$

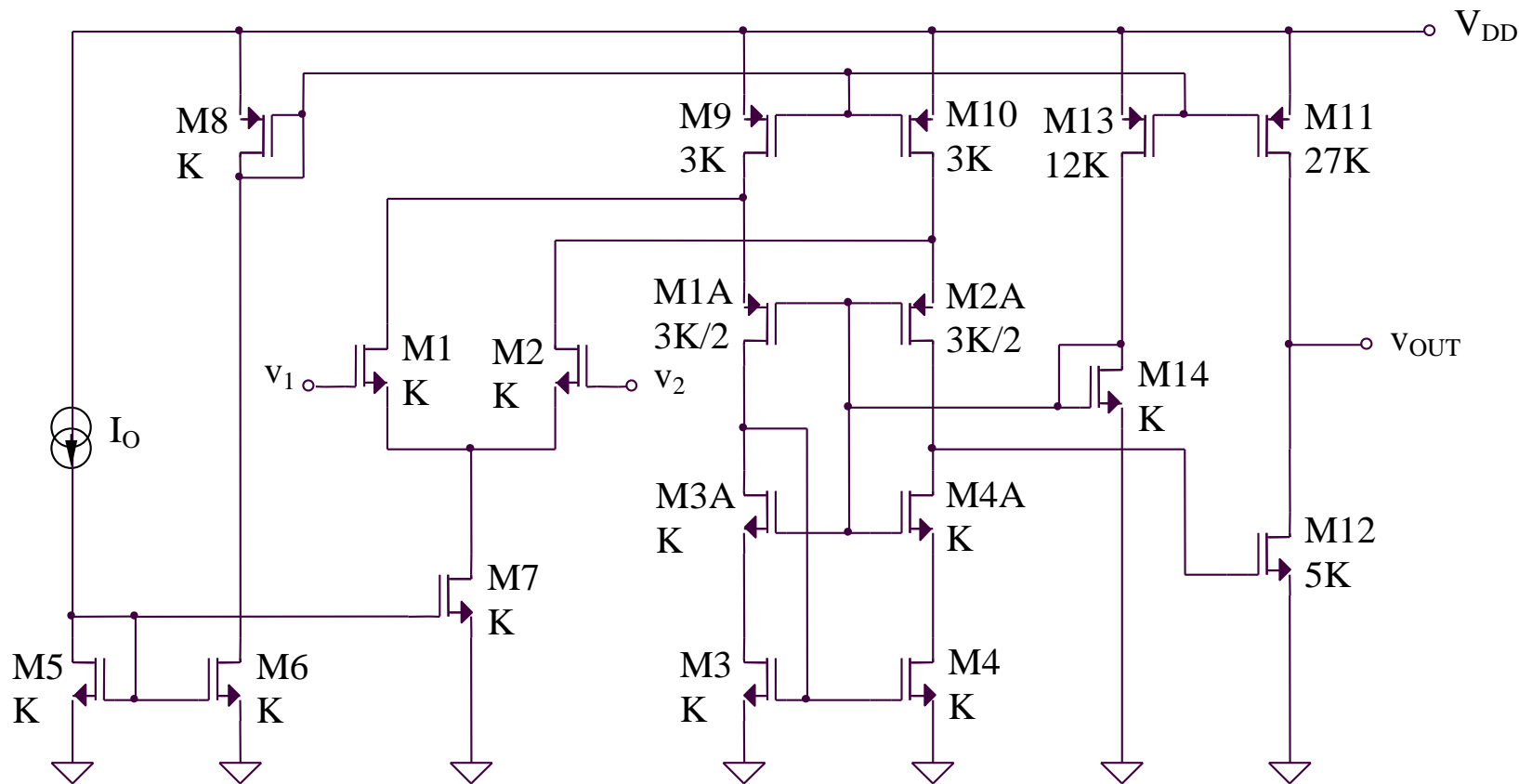
Regim dinamic

$$i_O = i_{D4} - i_{D8} = i_{D4} - i_{D3} = (I_{D6} - i_{D2}) - (I_{D5} - i_{D1}) = i_{D1} - i_{D2}$$

$$a = g_{m1} R_O$$

$$R_O = r_{ds8} g_{m8} r_{ds10} // [r_{ds4} g_{m4} (r_{ds6} // r_{ds2})]$$

6.3.12. Amplificator operational cascada intoarsa (3)

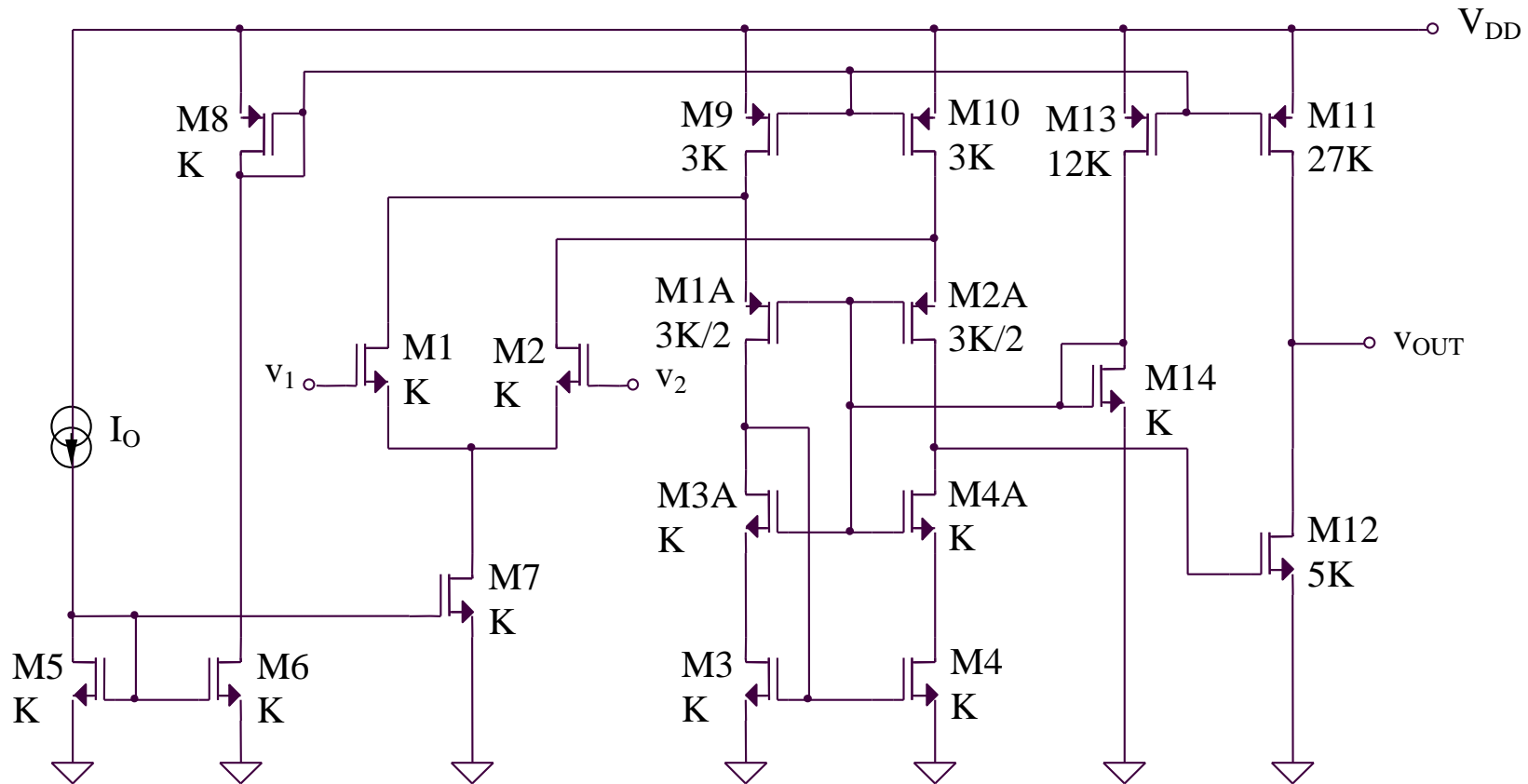


Regim static

$$I_{D5} = I_{D6} = I_{D7} = I_{D8} = I_O; \quad I_{D9} = I_{D10} = 3I_O; \quad I_{D13} = I_{D14} = 12I_O$$

$$I_{D11} = I_{D12} = 27I_O; \quad I_{D3A} = I_{D4A} = I_{D1A} = I_{D2A} = I_{D3} = I_{D4} = 3I_O - \frac{I_O}{2} = \frac{5I_O}{2}$$

6.3.12. Amplificator operational cascada intoarsa (3)



Regim dinamic

$$a_1 = g_{m1}R_{O1}$$

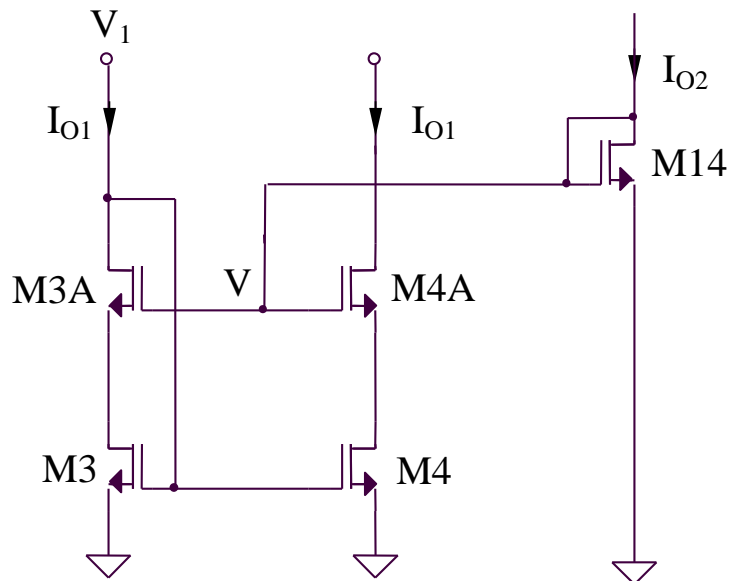
$$R_{O1} = g_{m4A}r_{ds4A}^2 // r_{ds2A} [1 + g_{m2A}(r_{ds10} // r_{ds2})]$$

$$a_2 = -g_{m12}R_{O2}$$

$$R_{O2} = r_{ds11} // r_{ds12}$$

6.3.12. Amplificator operational cascada intoarsa (3)

Analiza functionarii sursei de curent pentru tensiuni reduse



Tranzistoare identice

$$\begin{aligned} V_{GS3A} &= V_{GS4A} = \\ &= V_{GS3} = V_{GS4} = V_{GS} = \\ &= V_T + \sqrt{\frac{2I_{O1}}{K}} \end{aligned}$$

$$V = V_{GS14} = V_T + \sqrt{\frac{2I_{O2}}{K}}$$

$$V_{DS3} = V_{DS4} = V - V_{GS} > V_{GS} - V_T \Rightarrow V > 2V_{GS} - V_T$$

$$V_{DS3A} = V_{GS} - (V - V_{GS}) = 2V_{GS} - V > V_{GS} - V_T \Rightarrow V < V_{GS} + V_T$$

Se obtine:

$$\text{Deci: } 2\left(V_T + \sqrt{\frac{2I_{O1}}{K}}\right) - V_T < V_T + \sqrt{\frac{2I_{O2}}{K}} < \sqrt{\frac{2I_{O1}}{K}} + 2V_T$$

$$4I_{O1} < I_{O2} \quad \text{si} \quad \sqrt{\frac{2I_{O2}}{K}} - \sqrt{\frac{2I_{O1}}{K}} < V_T \quad - \text{conditii de proiectare}$$

Avantaj: $V_{1min} = V_{GS}$ (fata de oglinda clasica cascod avand $V_{1min} = 2V_{GS}$)

*Amplificatoare operationale transconductanta
(operational transconductance amplifiers – OTA)
in tehnologie CMOS*

Diferente fata de amplificatorul operational clasic:

- iesire in curent
- este utilizat in mod uzual in bucla deschisa, fara reactie negativa; amplitudinea tensiunii de iesire este controlata prin intermediul rezistentei de iesire

$$i_{OUT} = G_m(v_1 - v_2)$$

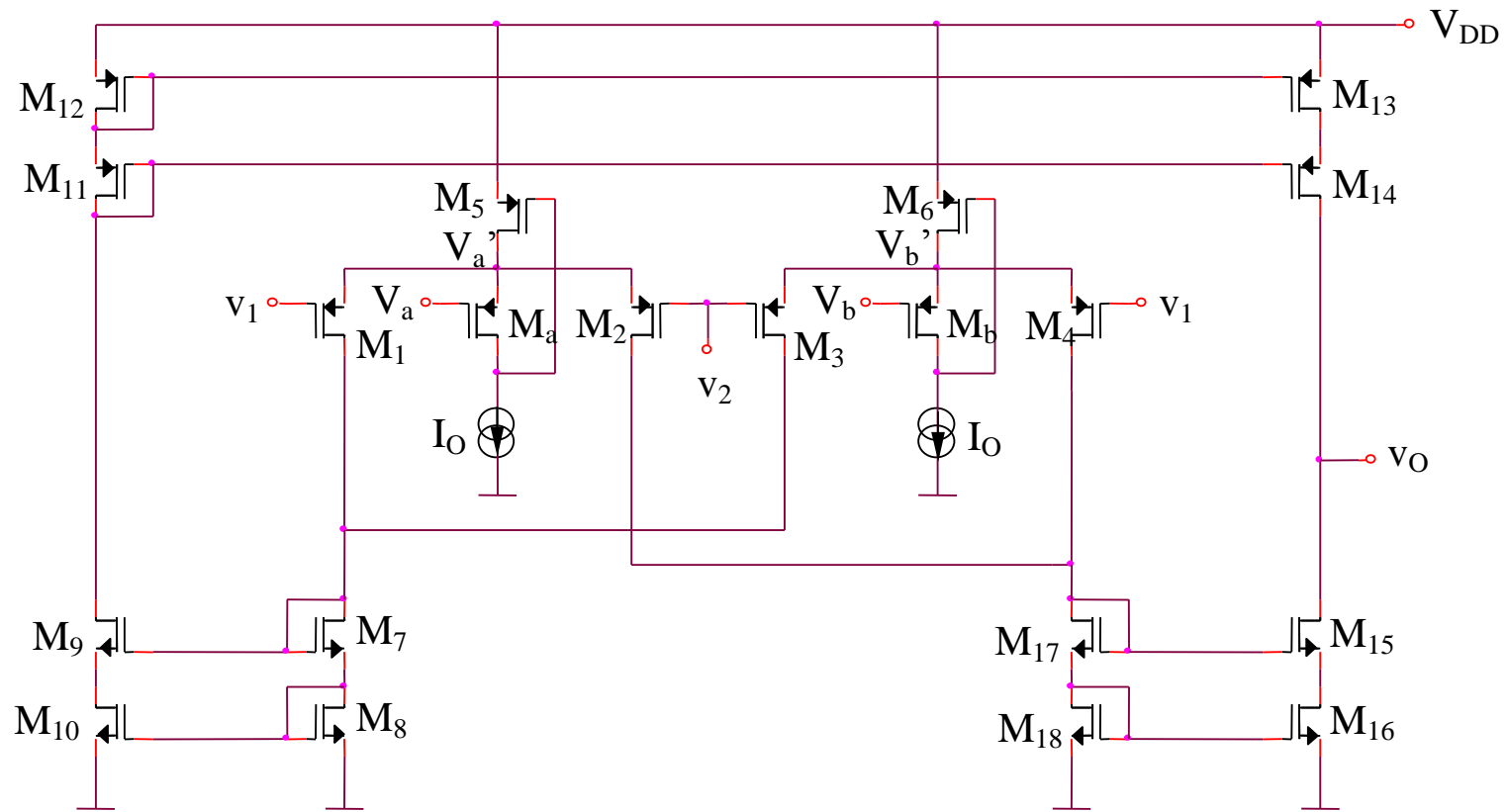
$$v_{OUT} = i_{OUT} R_O = G_m R_O (v_1 - v_2)$$

$$a = \frac{v_{OUT}}{v_1 - v_2} = G_m R_O$$

Dezavantaje:

- neliniaritatea amplificatorului transconductanta
- dependenta de temperatura a transconductantei echivalente

6.3.13. Amplificator operational transconductanta (1) - aplicatie AO

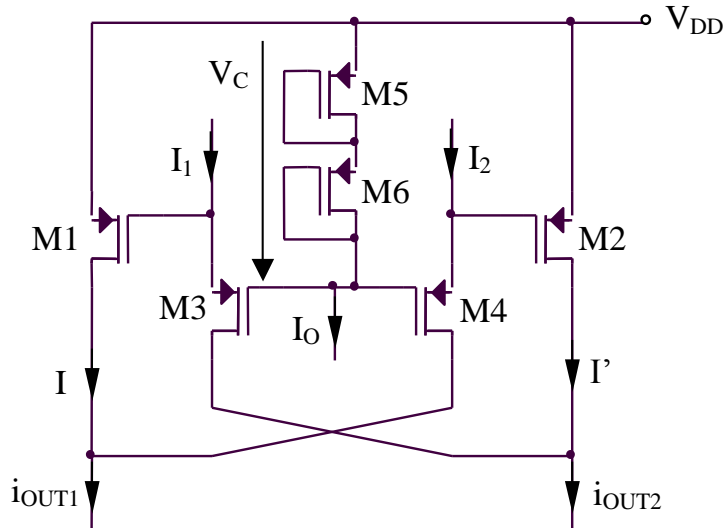


$$R_O = r_{ds14} g_{m14} r_{ds13} // r_{ds15} g_{m15} r_{ds16}$$

$$a = G_m R_O = \frac{1}{2} K (V_B - V_A) g_{m15} r_{ds15}^2$$

6.3.14. Amplificator operational transconductanta (2)

Circuit de extragere a radacinii patrate (I)



$$V_{SG1} + V_{SG3} = V_{SG5} + V_{SG6}$$

$$\sqrt{I} = \sqrt{\frac{K}{2}}(V_C - 2V_T) - \sqrt{I_1}$$

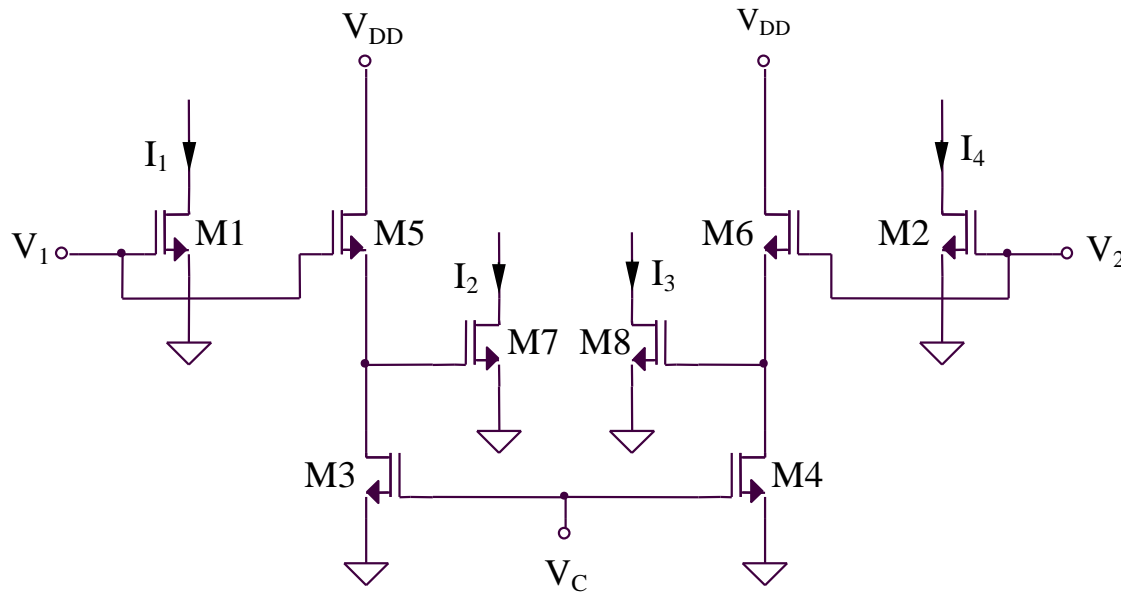
$$I = I_1 + \frac{K}{2}(V_C - 2V_T)^2 - \sqrt{2KI_1}(V_C - 2V_T)$$

$$i_{OUT1} = I + I_2 = I_1 + I_2 + \frac{K}{2}(V_C - 2V_T)^2 - \sqrt{2KI_1}(V_C - 2V_T)$$

$$i_{OUT2} = I_1 + I_2 + \frac{K}{2}(V_C - 2V_T)^2 - \sqrt{2KI_2}(V_C - 2V_T)$$

$$i_{OUT} = i_{OUT1} - i_{OUT2} = \sqrt{2K}(V_C - 2V_T)(\sqrt{I_2} - \sqrt{I_1})$$

6.3.16. Amplificator operational transconductanta (4)



$$I_1 - I_2 = \frac{K}{2} (V_1 - V_T)^2 - \frac{K}{2} (V_1 - V_C - V_T)^2 = \frac{K}{2} V_C (2V_1 - V_C - 2V_T)$$

$$I_4 - I_3 = \frac{K}{2} V_C (2V_2 - V_C - 2V_T)$$

$$i_{OUT} = (I_1 - I_2) - (I_4 - I_3) = KV_C (V_1 - V_2)$$