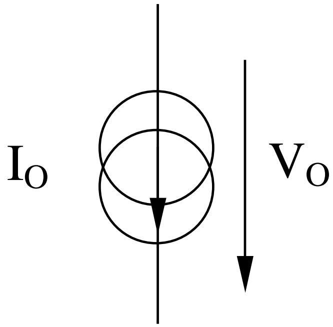
Chapter 2
Current and voltage sources

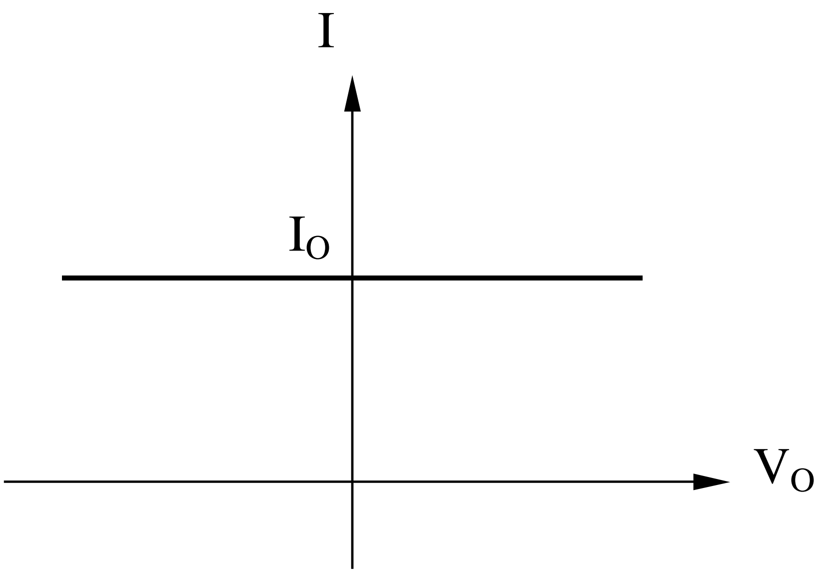
2.1. Current sources

2.1.1. Introduction

2.1. Current sources

2.1.1. Introduction





Parameters:

- The output current I_O is the current generated by the circuit [A]

- The output resistance [Ω]

$$R_O = \left.\frac{dV_O}{dI_O}\right|_{V_{CC},T=ct.}$$

- Output minimum voltage [V]
- Temperature coefficient [A/K]

$$TC_{I_O} = \left.\frac{dI_O}{dT}\right|_{R_L, V_{CC}=ct.}$$

- Relative temperature coefficient [1/K]

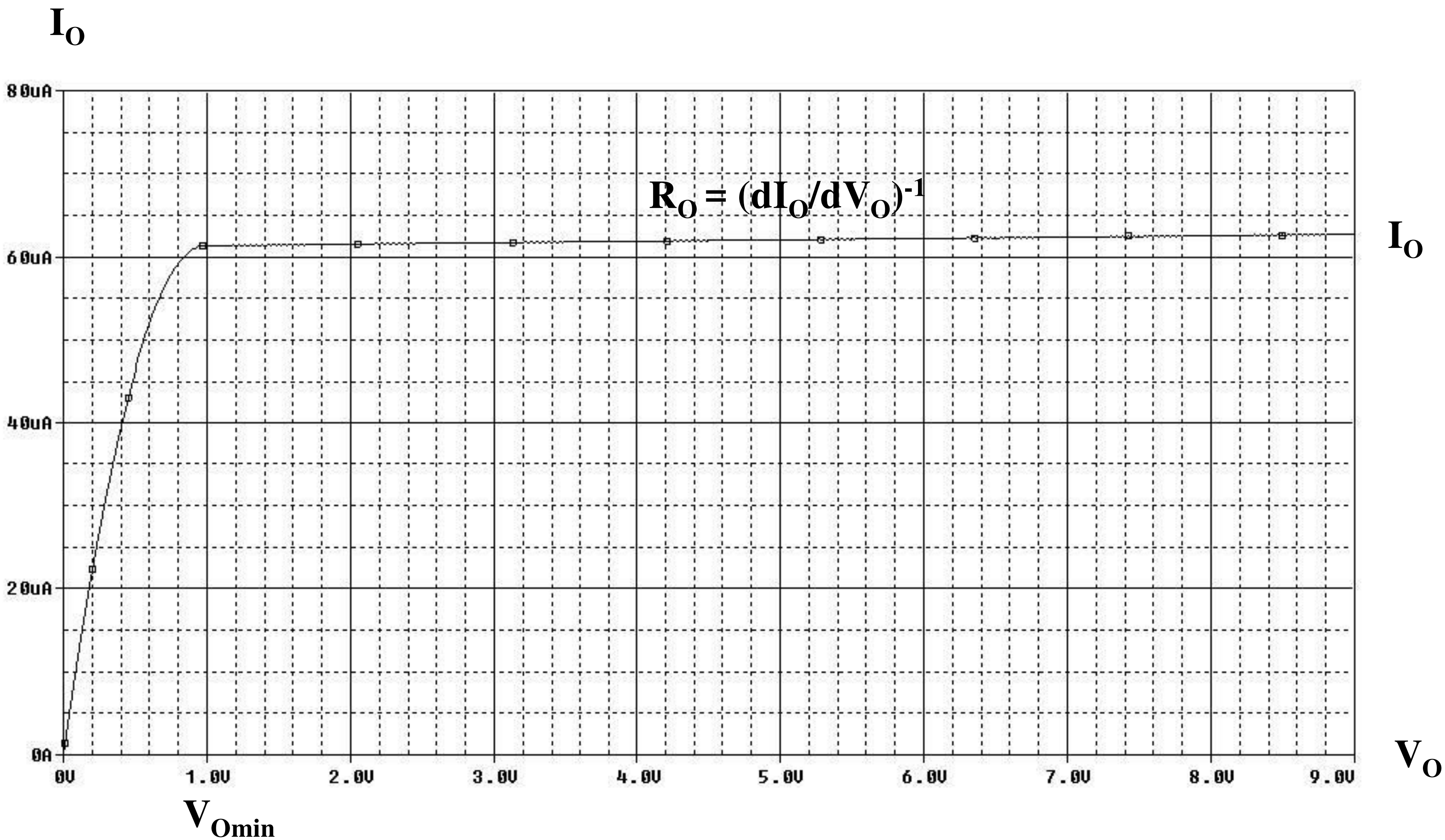
$$RTC_{I_O} = \left.\frac{1}{I_O}\frac{dI_O}{dT}\right|_{R_L, V_{CC}=ct.}$$

- Power supply rejection ratio [A/V]

$$PSRR = \left.\frac{dI_O}{dV_{CC}}\right|_{R_L, T=ct.}$$

- Sensibility of the output current on supply voltage variations [-]

$$S_{V_{CC}}^{I_O} = \left.\frac{dI_O / I_O}{dV_{CC} / V_{CC}}\right|_{R_L, T=ct.} = \left.\frac{V_{CC}}{I_O}\frac{dI_O}{dV_{CC}}\right|_{R_L, T=ct.}$$



Output characteristic of a current source

Classification

I. Elementary current sources
- reduced complexity
- poor performances

II. Cascode current source
- increased output resistance
- increased minimum output voltage
- increased minimum supply voltage

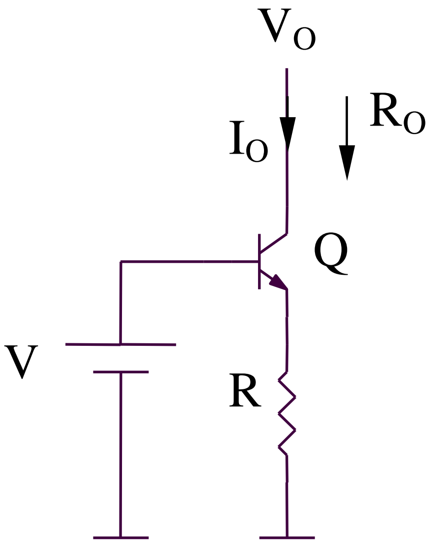
III. Self-biased current sources
- reduced dependence I_O (V_{CC})
- requires a starting circuit

IV. Temperature-compensated current sources
- increased complexity

2.1.2. Elementary current sources

2.1.2. Elementary current sources

Bipolar current source with a transistor

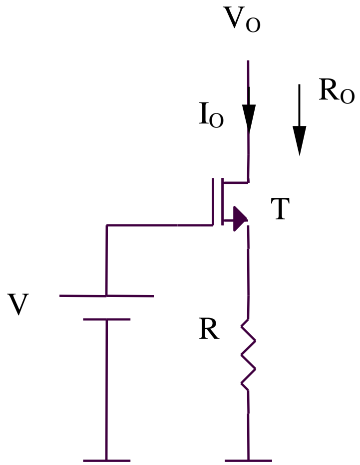


$$I_O = \frac{V - V_{BE}}{R}$$

$$R_O = r_o\left(1 + \frac{\beta R}{r_\pi + R}\right)$$

$$V_{O\min} = V - V_{BE} + V_{CEsat}$$

MOS current source with a transistor



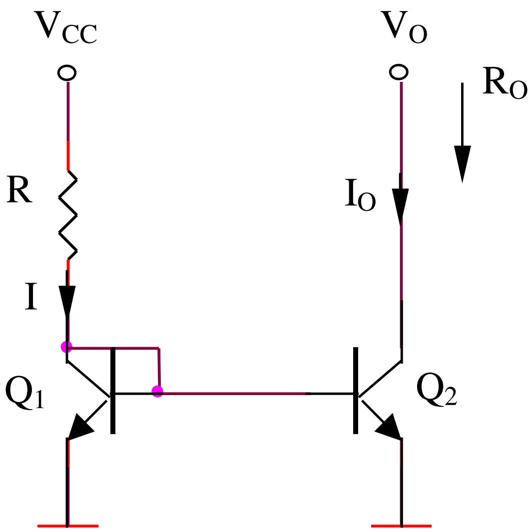
$$\left.\begin{aligned} V &= V_{GS} + I_O R \\ I_O &= \frac{K}{2}\left(V_{GS} - V_T\right)^2 \end{aligned}\right\} \Rightarrow V = V_{GS} + \frac{KR}{2}\left(V_{GS} - V_T\right)^2$$

$$\Rightarrow V_{GS}\left(> V_T\right) \Rightarrow I_O$$

$$R_O = r_{ds}\left(1 + g_m R\right)$$

$$V_{O\min} = V - V_{GS} + \left(V_{GS} - V_T\right) = V - V_T$$

Bipolar current mirror



Output current

$$I = \frac{V_{CC} - V_{BE}}{R} \cong I_{S1} \exp\left(\frac{V_{BE1}}{V_{th}}\right)$$

$$I_O \cong I_{S2} \exp\left(\frac{V_{BE2}}{V_{th}}\right)$$

$$V_{BE1} = V_{BE2}$$

$$\Rightarrow \quad \frac{I_O}{I} \cong \frac{I_{S2}}{I_{S1}} \quad \Rightarrow \quad I_O \cong \frac{V_{CC} - V_{BE}}{R} \frac{I_{S2}}{I_{S1}}$$

Output resistance

$$R_O = r_o = \frac{V_A}{I_{C2}} = \frac{V_A}{I_O}$$

Minimum output voltage

$$V_{O\min} = V_{CE2sat}.$$

Early effect

$$I = \frac{V_{CC} - V_{BE}}{R} = I_{S1}\exp\left(\frac{V_{BE1}}{V_{th}}\right)\left(1 + \frac{V_{CE1}}{V_A}\right)$$

$$I_O = I_{S2}\exp\left(\frac{V_{BE2}}{V_{th}}\right)\left(1 + \frac{V_{CE2}}{V_A}\right)$$

$$\frac{I_O}{I} = \frac{I_{S2}}{I_{S1}}\frac{1 + \frac{V_{CE1}}{V_A}}{1 + \frac{V_{CE2}}{V_A}} = \frac{I_{S2}}{I_{S1}}\frac{1 + \frac{V_{BE1}}{V_A}}{1 + \frac{V_O}{V_A}}$$

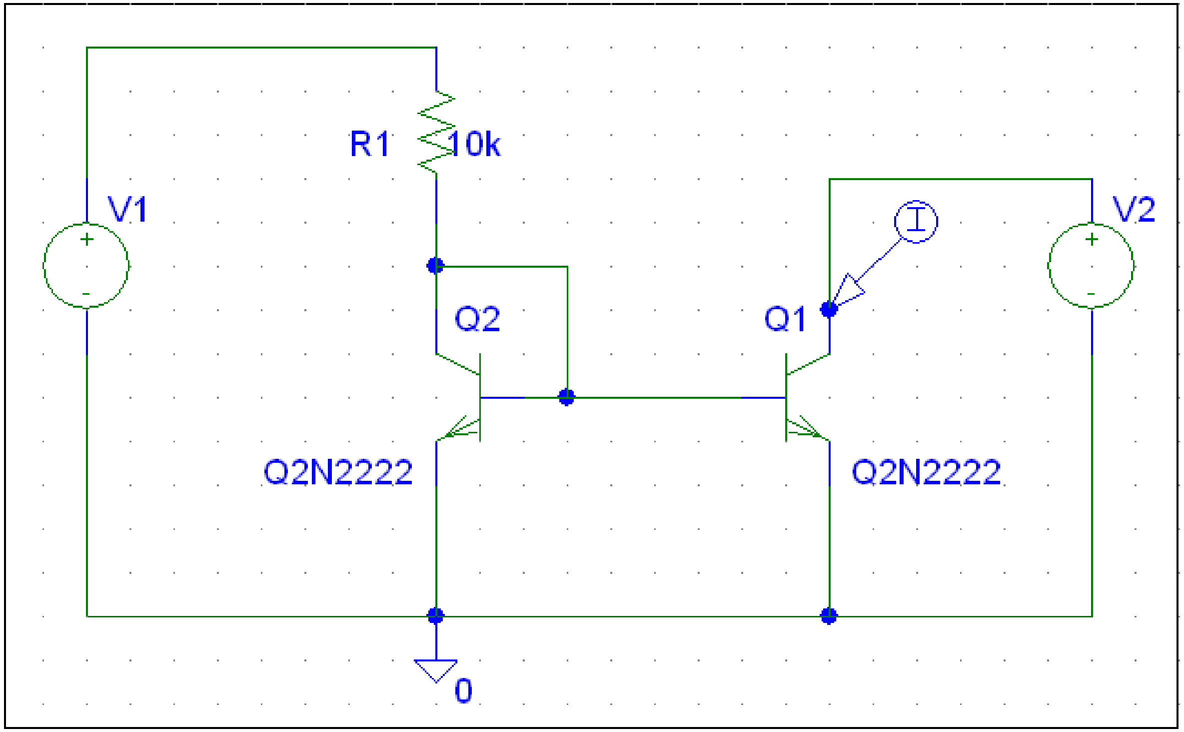
The influence of β

$$\frac{I_O}{I} = \frac{\beta I_B}{\beta I_B + 2I_B} = \frac{\beta}{\beta + 2}$$

SIMULATIONS for bipolar current mirror Output characteristic

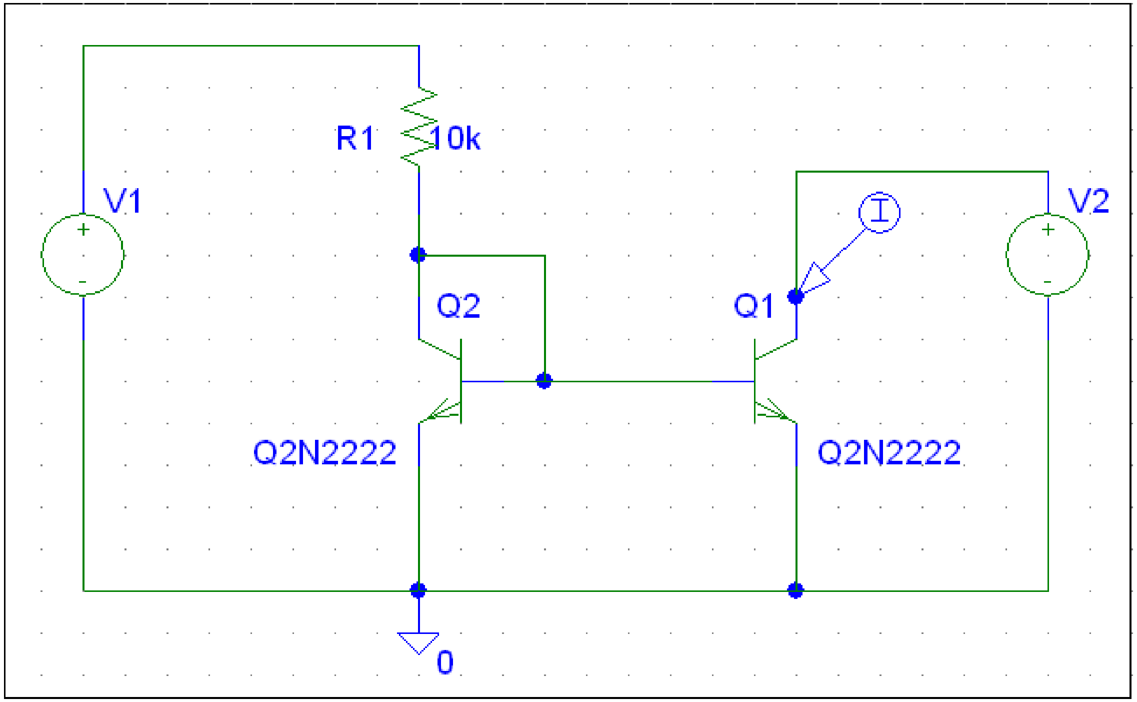
SIMULATIONS for bipolar current mirror
Output characteristic

SIM 2.1: I_{C2} (V2)



SIMULATIONS for bipolar current mirror
Output characteristic

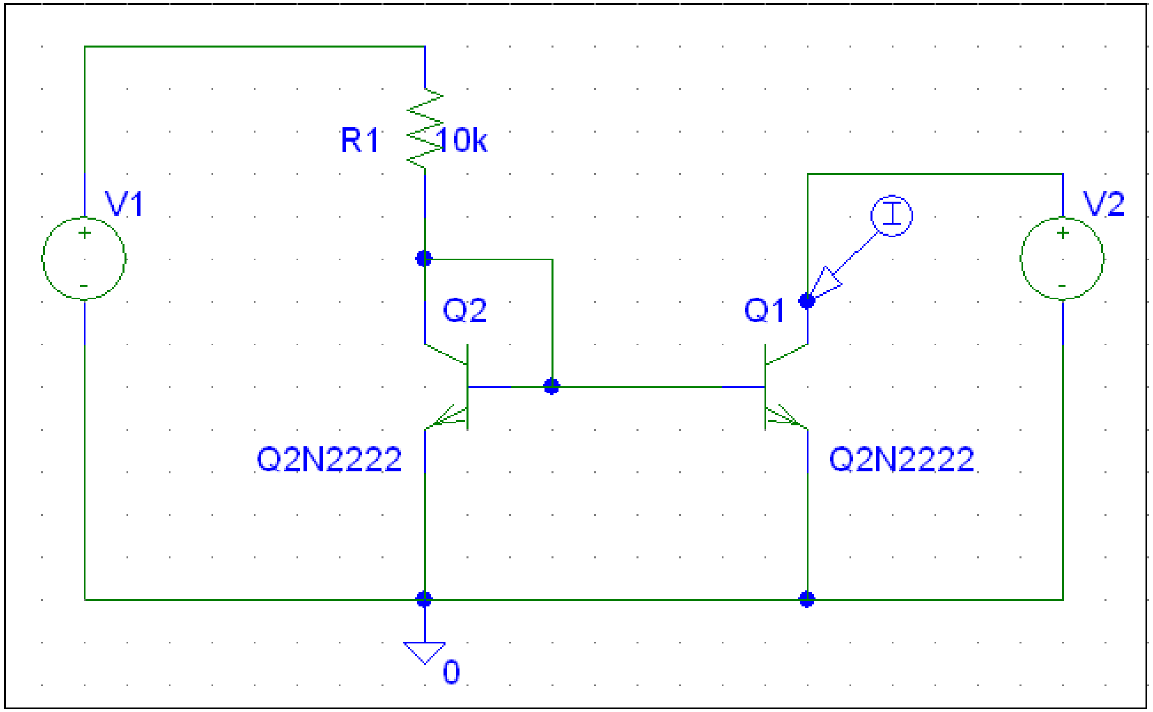
SIM 2.2: I_{C2} (V2), V_{A1} - parameter



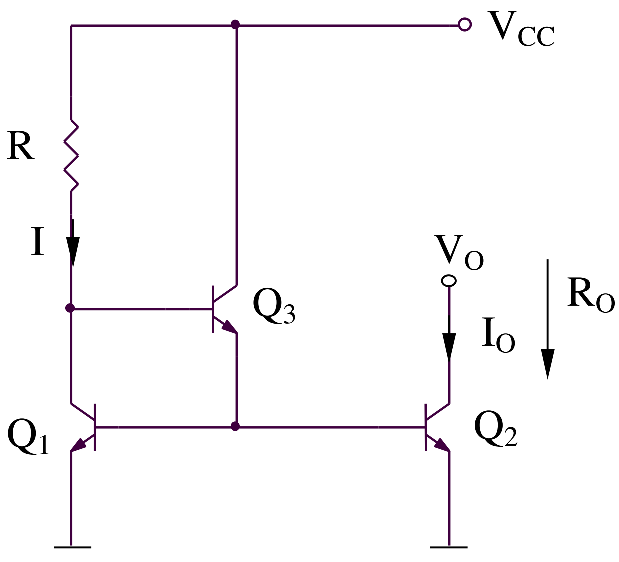
SIMULATIONS for bipolar current mirror
Dependence of the output current on the supply voltage

SIMULATIONS for bipolar current mirror
Dependence of the output current on the supply voltage

SIM 2.3: I_{C2} (V1)



Current mirror with reduction of influence of β (1)



Output current

$$I_O \cong I = \frac{V_{CC} - 2V_{BE}}{R}$$

Output resistance

$$R_O = r_o = \frac{V_A}{I_{C2}} = \frac{V_A}{I_O}$$

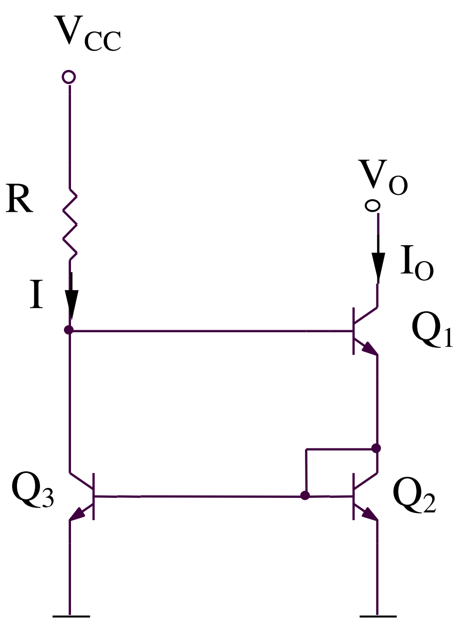
Minimum output voltage

$$V_{O\min} = V_{CE2sat}.$$

The influence of β

$$\frac{I_O}{I} = \frac{\beta I_B}{\beta I_B + \frac{2I_B}{\beta+1}} = \frac{1}{1 + \frac{2}{\beta^2 + \beta}} \cong 1$$

Current mirror with reduction of influence of β (2)



Output resistance

$$I_O \cong I = \frac{V_{CC} - 2V_{BE}}{R}$$

Minimum output voltage

$$R_O \cong \frac{\beta r_{o1}}{2}$$

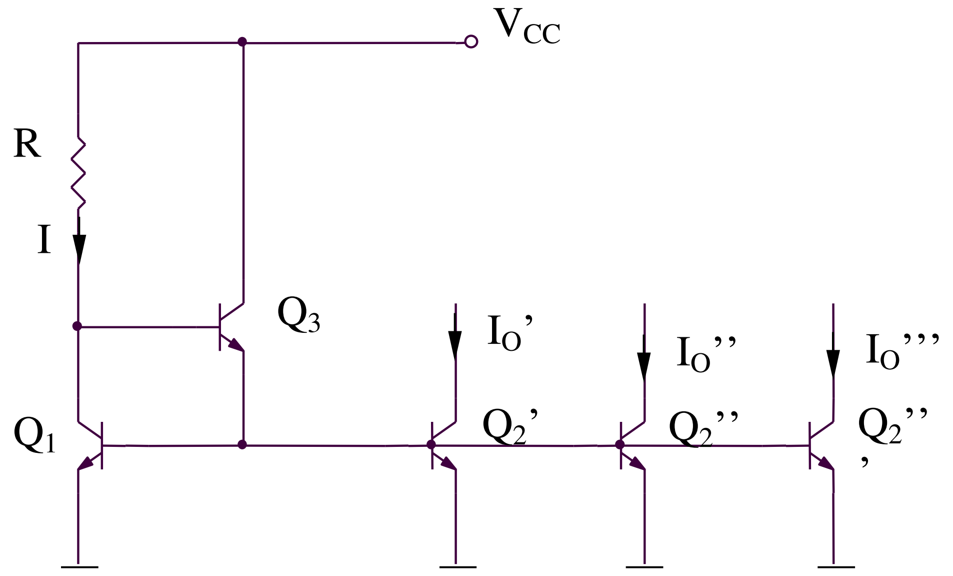
Early effect

$$V_{O\min} = V_{BE2} + V_{CE1sat}.$$

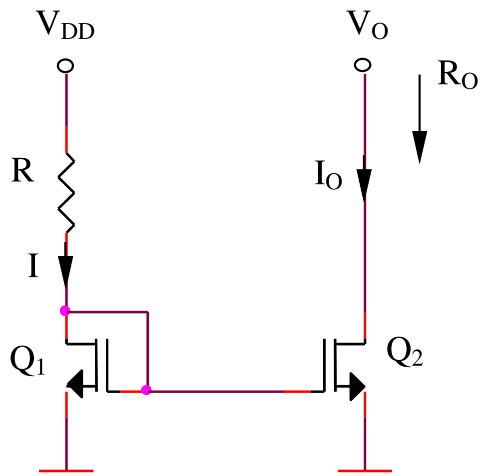
The influence of β

$$\frac{I_O}{I} = \frac{\frac{\beta(\beta+2)}{\beta+1} I_B}{\beta I_B + \frac{\beta+2}{\beta+1} I_B} = \frac{1}{1 + \frac{2}{\beta^2 + 2\beta}} \cong 1$$

Multiple current mirror with reduction of influence of β



MOS current mirror



Output current

$$\left.\begin{aligned} V_{DD} &= I_O R + V_{GS1} \\ I_O &= \frac{K}{2}\left(V_{GS1} - V_T\right)^2 \end{aligned}\right\} \Rightarrow V_{DD} = \frac{KR}{2}\left(V_{GS1} - V_T\right)^2 + V_{GS1} \Rightarrow$$

$$\Rightarrow \left(V_{GS1}\right)_{1,2} = V_T - \frac{1}{KR} \pm \frac{1}{KR}\sqrt{1 + 2KR(V_{DD} - V_T)}$$

As V_{GS} must be greater than V_T, it results:

$$V_{GS1} = V_T - \frac{1}{KR} + \frac{1}{KR}\sqrt{1 + 2KR(V_{DD} - V_T)}$$

$$\Rightarrow I_O = \frac{1}{KR^2}\left[1 + KR(V_{DD} - V_T) - \sqrt{1 + 2KR(V_{DD} - V_T)}\right]$$

Output resistance

$$R_O = r_{ds2} = \frac{1}{\lambda I_O}$$

Minimum output voltage

$$V_{O\min} = V_{DS2sat} = V_{GS2} - V_T = \sqrt{\frac{2I_O}{K}}$$

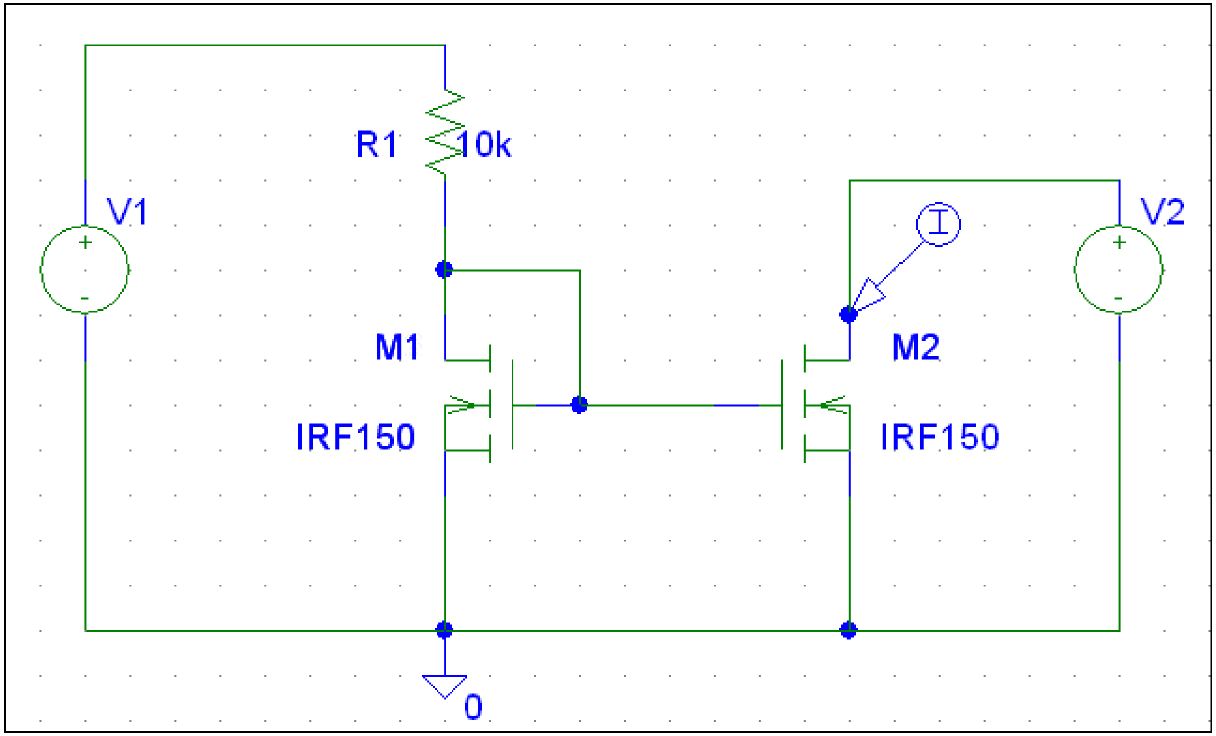
The effect of channel-length modulation

$$\frac{I_O}{I} = \frac{\frac{K}{2}(V_{GS2} - V_T)^2(1 + \lambda V_{DS2})}{\frac{K}{2}(V_{GS1} - V_T)^2(1 + \lambda V_{DS1})} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = \frac{1 + \lambda V_O}{1 + \lambda V_{GS1}}$$

SIMULATIONS for CMOS current mirror
Output characteristic

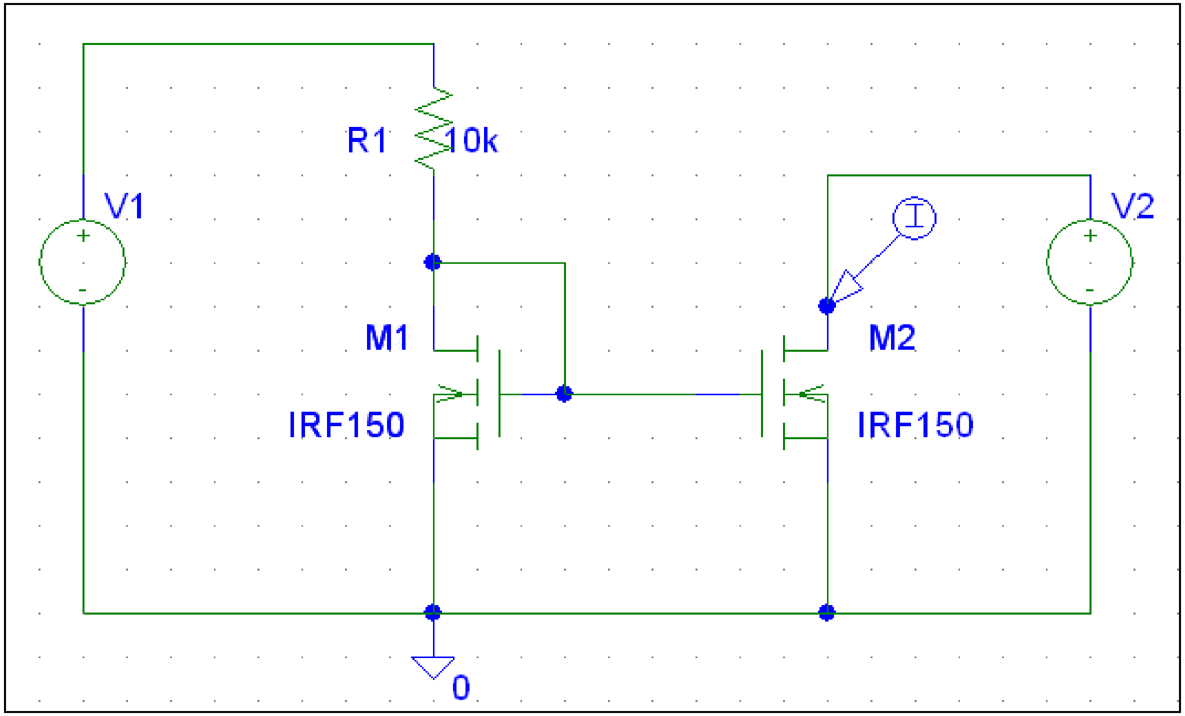
SIMULATIONS for CMOS current mirror
Output characteristic

SIM 2.4: I_{D2} (V2)



SIMULATIONS for CMOS current mirror
Output characteristic

SIM 2.5: I_{D2} (V2), r_{ds2} - parameter

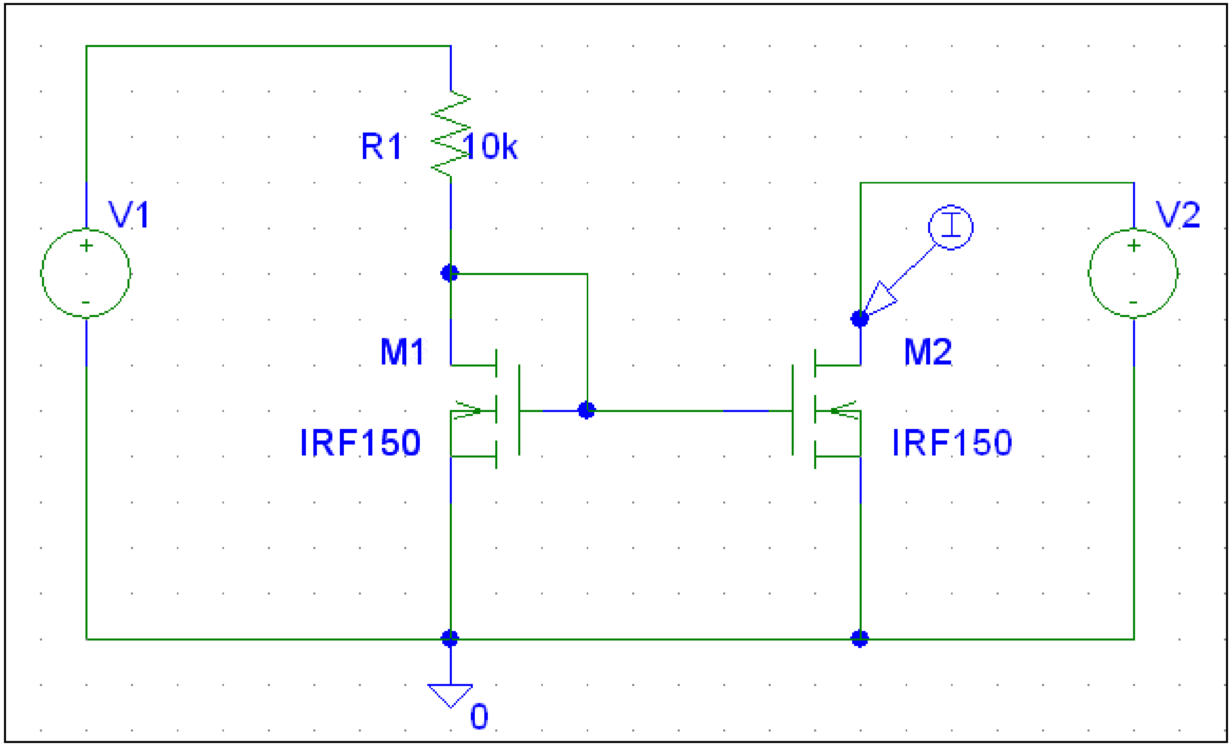


SIMULATIONS for CMOS current mirror

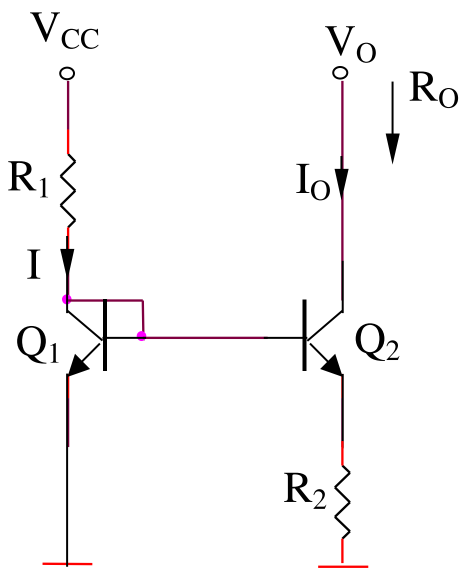
Dependence of the output current on the supply voltage

SIMULATIONS for CMOS current mirror
Dependence of the output current on the supply voltage

SIM 2.6: I_{D2} (V1)



Bipolar Widlar current source



Output current

$$I = \frac{V_{CC} - V_{BE}}{R_1}$$

$$I_O = \frac{V_{BE1} - V_{BE2}}{R_2} = \frac{V_{th} \ln\left(\frac{I}{I_S}\right) - V_{th} \ln\left(\frac{I_O}{I_S}\right)}{R_2}$$

$$I_O = \frac{V_{th}}{R_2} \ln\left(\frac{I}{I_O}\right) = \frac{V_{th}}{R_2} \ln\left(\frac{V_{CC} - V_{BE}}{R_1 I_O}\right)$$

Minimum output voltage

$$V_{O\min} = V_{CE2sat.} + I_O R_2$$

Output resistance

$$R_O = r_o\left(1 + \frac{\beta R_2}{r_{\pi 2} + R_2 + (1/g_{m1})//R_1}\right) = \frac{V_A}{I_O}\left(1 + \frac{\beta R_2}{r_{\pi 2} + R_2 + (1/g_{m1})//R_1}\right)$$

Power supply rejection ratio

$$\frac{dI_O}{dV_{CC}} = \frac{d}{dV_{CC}}\left[\frac{V_{th}}{R_2}\ln\left(\frac{V_{CC}-V_{BE}}{R_1 I_O}\right)\right]$$

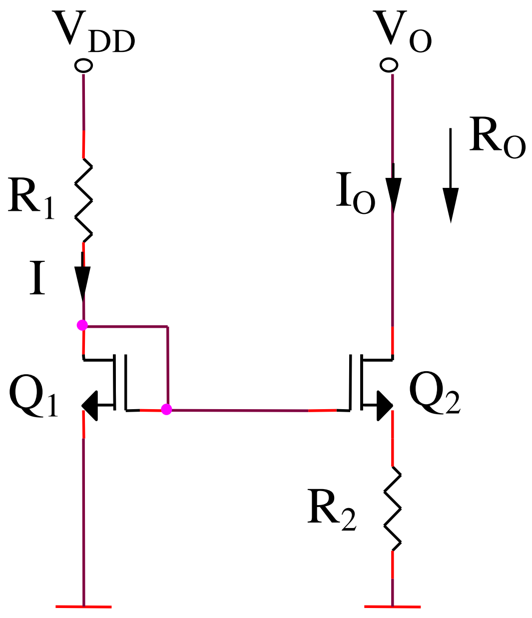
$$\frac{dI_O}{dV_{CC}} = \frac{V_{th}}{R_2}\frac{R_1 I_O}{V_{CC}-V_{BE}}\frac{R_1 I_O-\left(V_{CC}-V_{BE}\right)R_1\frac{dI_O}{dV_{CC}}}{\left(R_1 I_O\right)^2}$$

$$\frac{dI_O}{dV_{CC}} = \frac{\frac{1}{R_2}\frac{V_{th}}{V_{CC}-V_{BE}}}{1+\frac{V_{th}}{R_2 I_O}}$$

Sensibility of the output current on supply voltage variations

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{dI_O}{dV_{CC}} = \frac{1}{1+\frac{R_2 I_O}{V_{th}}} = \frac{1}{1+\ln\left(\frac{V_{CC}-V_{BE}}{R_1 I_O}\right)}$$

MOS Widlar current source



Output current

$$V_{GS1} = V_T - \frac{1}{KR_1} + \frac{1}{KR_1}\sqrt{1 + 2KR_1(V_{DD} - V_T)}$$

$$V_{GS1} = V_{GS2} + I_O R_2 = V_{GS2} + \frac{KR_2}{2}(V_{GS2} - V_T)^2$$

$$(V_{GS2} > V_T)$$

$$I_O = \frac{K}{2}(V_{GS2} - V_T)^2 (1 + \lambda V_{DS2})$$

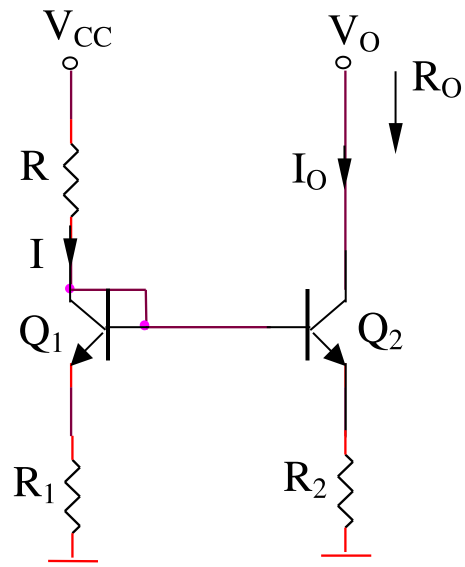
Minimum output voltage

$$V_{O\min} = V_{DS2sat} + I_O R_2 = \sqrt{\frac{2I_O}{K}} + I_O R_2$$

Output resistance

$$R_O = r_{ds2}(1 + g_{m2} R_2)$$

Standard current source



Output current

$$v_{BE1} + R_1 I = v_{BE2} + R_2 I_O$$

$$I_O = \frac{1}{R_2}\left(R_1 I + v_{BE1} - v_{BE2}\right)$$

$$\frac{I_O}{I} = \frac{R_1}{R_2} + \frac{V_{th}}{R_2 I}\ln\left(\frac{I}{I_O}\frac{I_{S2}}{I_{S1}}\right)$$

It is possible to determine I/I_O because:

$$I = \frac{V_{CC} - v_{BE}}{R + R_1}$$

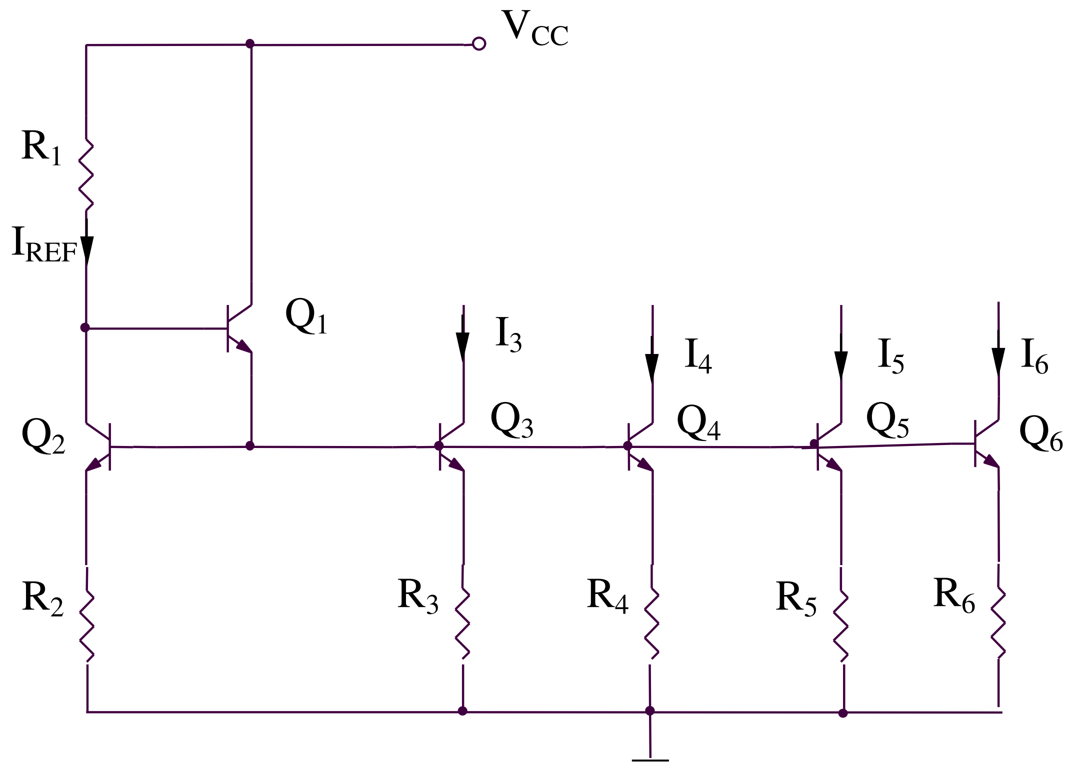
If $R_1 I \gg v_{BE1} - v_{BE2}$:

$$\frac{I_O}{I} = \frac{R_1}{R_2}$$

Output resistance

$$R_O = r_{o2}\left(1 + \frac{\beta R_2}{R_2 + r_{\pi 2} + R // \left(1/g_{m1} + R_1\right)}\right)$$

Standard current sources with multiple outputs



If the emitters areas are choose in order to have equal courant densities *j*, the base-emitter voltages are also equal.

$$v_{BE2} - v_{BE3} = V_{th} \ln\left(\frac{I_{REF}}{I_3} \frac{I_{S3}}{I_{S2}} \right) = V_{th} \ln\left(\frac{jA_2}{jA_3} \frac{A_3}{A_2} \right) = 0$$

So:

$$v_{BE2} = \ldots = v_{BE6}$$

and:

$$I_3 R_3 = I_4 R_4 = I_5 R_5 = I_6 R_6 = I_{REF} R_2$$

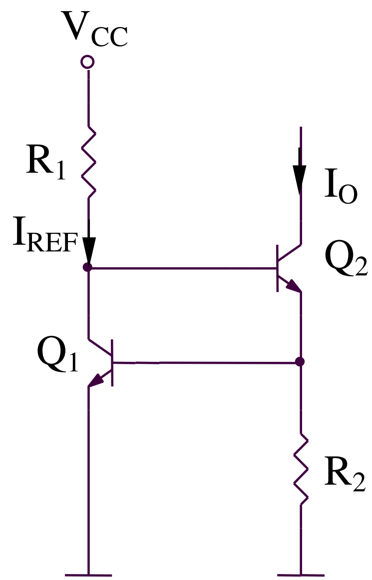
The four output currents are:

$$I_3 = I_{REF} \frac{R_2}{R_3}; \ldots ; I_6 = I_{REF} \frac{R_2}{R_6}$$

where:

$$I_{REF} = \frac{V_{CC} - 2v_{BE}}{R_1 + R_2}$$

Current source using as reference the base-emitter voltage



$$I_O = \frac{v_{BE1}}{R_2} = \frac{V_{th}}{R_2} \ln \frac{V_{CC} - 2v_{BE}}{R_1 I_S}$$

2.1.3. Cascode current sources

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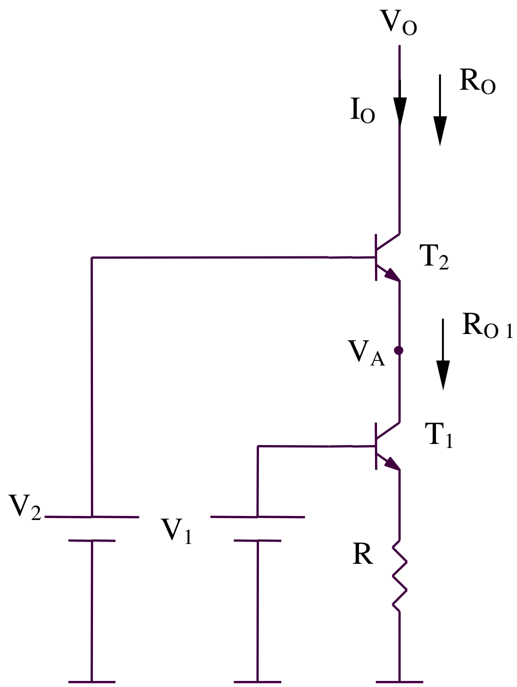
Bipolar cascode current source (1)



Output current

$$I_O = \frac{V_1 - V_{BE1}}{R}$$

Output resistance

$$R_O = r_{o2}\left(1 + \frac{\beta R_{O1}}{r_{\pi 2} + R_{O1}}\right) \cong \beta r_{O2}$$

$$R_{O1} = r_{o1}\left(1 + \frac{\beta R}{r_{\pi 1} + R}\right) >> r_{\pi 2}$$

Minimum output voltage

$$V_{O\min} = V_A + V_{CE2sat} = V_2 - V_{BE2} + V_{CE2sat}$$

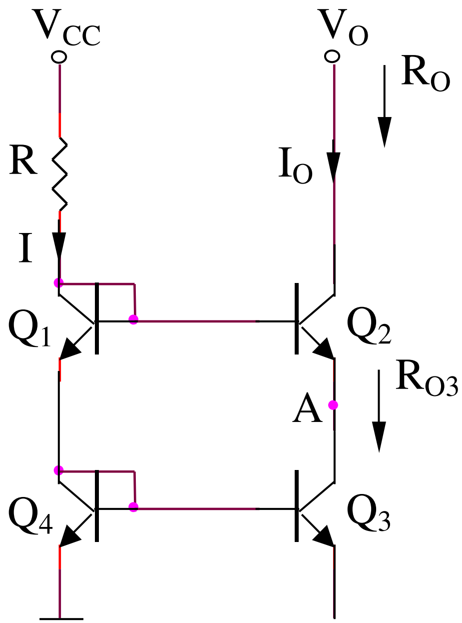
It is necessary that:

$$V_{CE1} > V_{CE1sat} \Leftrightarrow$$

$$\Leftrightarrow (V_2 - V_{BE2}) - (V_1 - V_{BE1}) > V_{CE1sat} \Leftrightarrow$$

$$\Leftrightarrow V_2 - V_1 > V_{CE1sat}$$

Bipolar cascode current source (2)



Output current

$$I_O = I = \frac{V_{CC} - 2v_{BE}}{R}$$

Output resistance

$$R_O = r_{o2}\left(1 + \beta \frac{R_{O3}}{r_{\pi 2} + R_{O3} + R // (2 / g_{m1})}\right)$$

$$R_{O3} = r_{o3} >> r_{\pi 2}, R // (2 / g_{m1})$$

So:

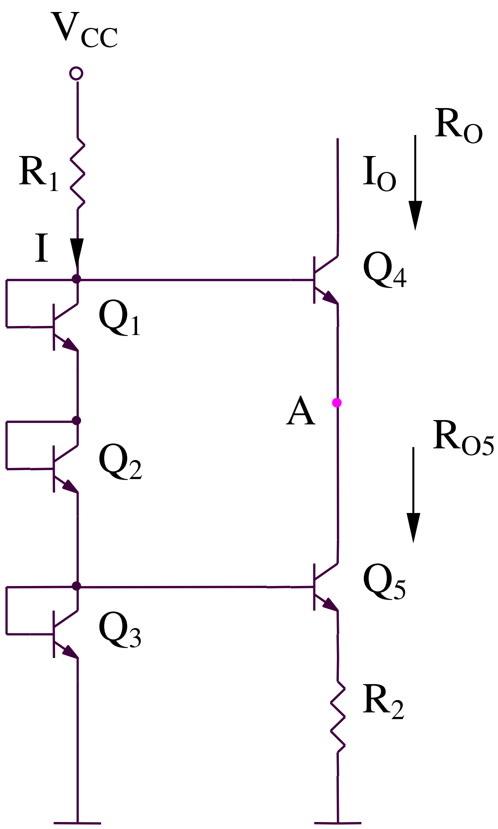
$$R_O \cong \beta r_{o2}$$

Minimum output voltage

$$V_{O\min} = V_A + V_{CE2sat}$$

$$V_A = v_{BE1} + v_{BE4} - v_{BE2} = v_{BE}$$

Bipolar cascode current source (3)



Output current

$$I_O = \frac{v_{BE3} - v_{BE5}}{R_2} = \frac{V_{th}}{R_2}\ln\left(\frac{I}{I_O}\right)$$

$$I = \frac{V_{CC} - 3v_{BE}}{R_1}$$

Output resistance

$$R_O = r_{o4}\left(1 + \beta\frac{R_{O5}}{r_{\pi 4} + R_{O5} + R_1 // (3/g_{m1})}\right)$$

$$R_{O5} \cong r_{o5}\left(1 + \frac{\beta R_2}{r_{\pi 5} + R_2 + 1/g_{m3}}\right)$$

$$R_{O5} >> r_{\pi 4}, R_1 // (3/g_{m1})$$

So:

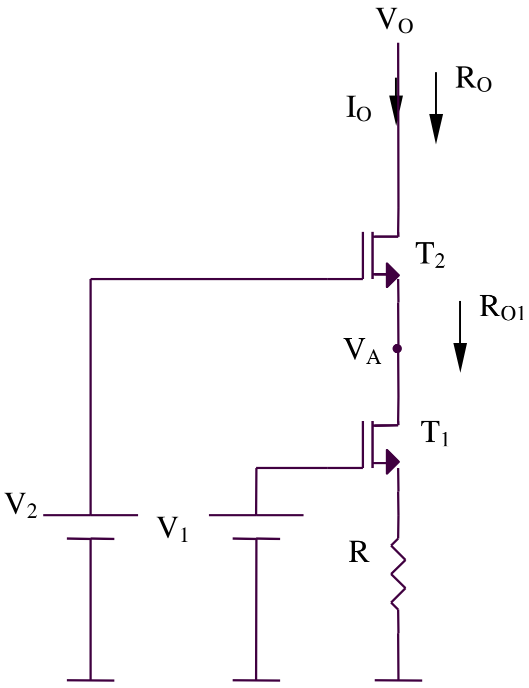
$$R_O \cong \beta r_{o4}$$

Minimum output voltage

$$V_{O\min} = V_A + V_{CE4sat}$$

$$V_A = 2v_{BE}$$

MOS cascode current source (1)



Output current

$$\left.\begin{aligned} V_1 &= V_{GS1} + I_O R \\ I_O &= \frac{K}{2}\left(V_{GS1} - V_T\right)^2 \end{aligned}\right\} \Rightarrow V_1 = V_{GS1} + \frac{KR}{2}\left(V_{GS1} - V_T\right)^2$$

$$\Rightarrow V_{GS1}\left(> V_T\right) \Rightarrow I_O$$

Output resistance

$$R_O = r_{ds2}\left(1 + g_m R_{O1}\right) \cong g_m r_{ds}^2$$

$$R_{O1} = r_{ds1}\left(1 + g_m R\right)$$

Minimum output voltage

$$V_{O\,min} = V_2 - V_{GS2} + \left(V_{GS2} - V_T\right) = V_2 - V_T$$

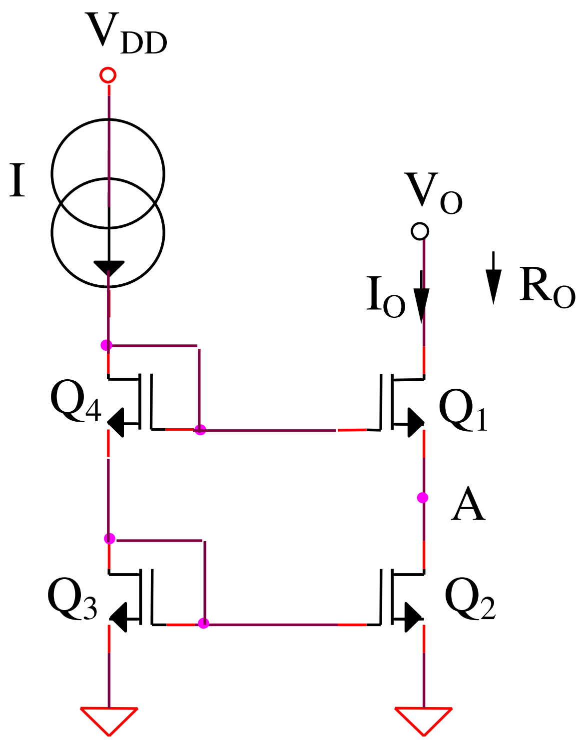
It is necessary that:

$$V_{DS1} > V_{DS1sat} \Leftrightarrow$$

$$\Leftrightarrow \left(V_2 - V_{GS2}\right) - \left(V_1 - V_{GS1}\right) > V_{DS1sat} \Leftrightarrow$$

$$\Leftrightarrow V_2 - V_1 > V_{DS1sat} = V_{GS} - V_T = \sqrt{\frac{2I_O}{K}}$$

MOS cascode current source (2)



Output current

$$\frac{I_O}{I} = \frac{1+\lambda V_{DS2}}{1+\lambda V_{DS3}}$$

Output resistance

$$R_O = r_{ds1}\left(1+g_{m1}r_{ds2}\right) \cong g_m r_{ds}^2$$

Minimum output voltage

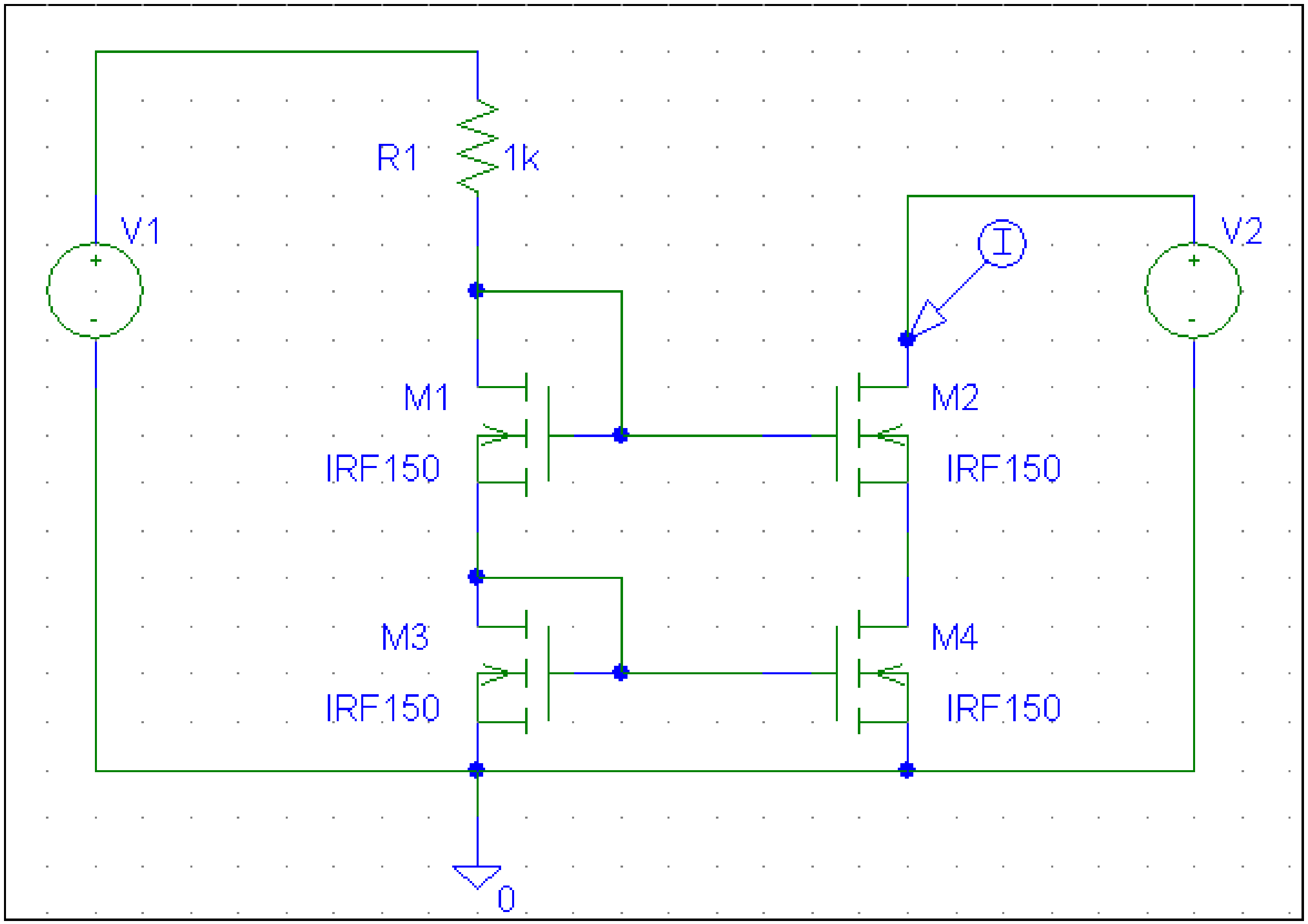
$$V_{O\min} = V_A + V_{DS1sat} = V_{GS} + \left(V_{GS} - V_T\right)$$

$$V_{O\min} = 2V_{GS} - V_T \cong V_T + 2\sqrt{\frac{2I}{K}}$$

SIMULATIONS for CMOS cascode current mirror Output characteristic

SIMULATIONS for CMOS cascode current mirror Output characteristic

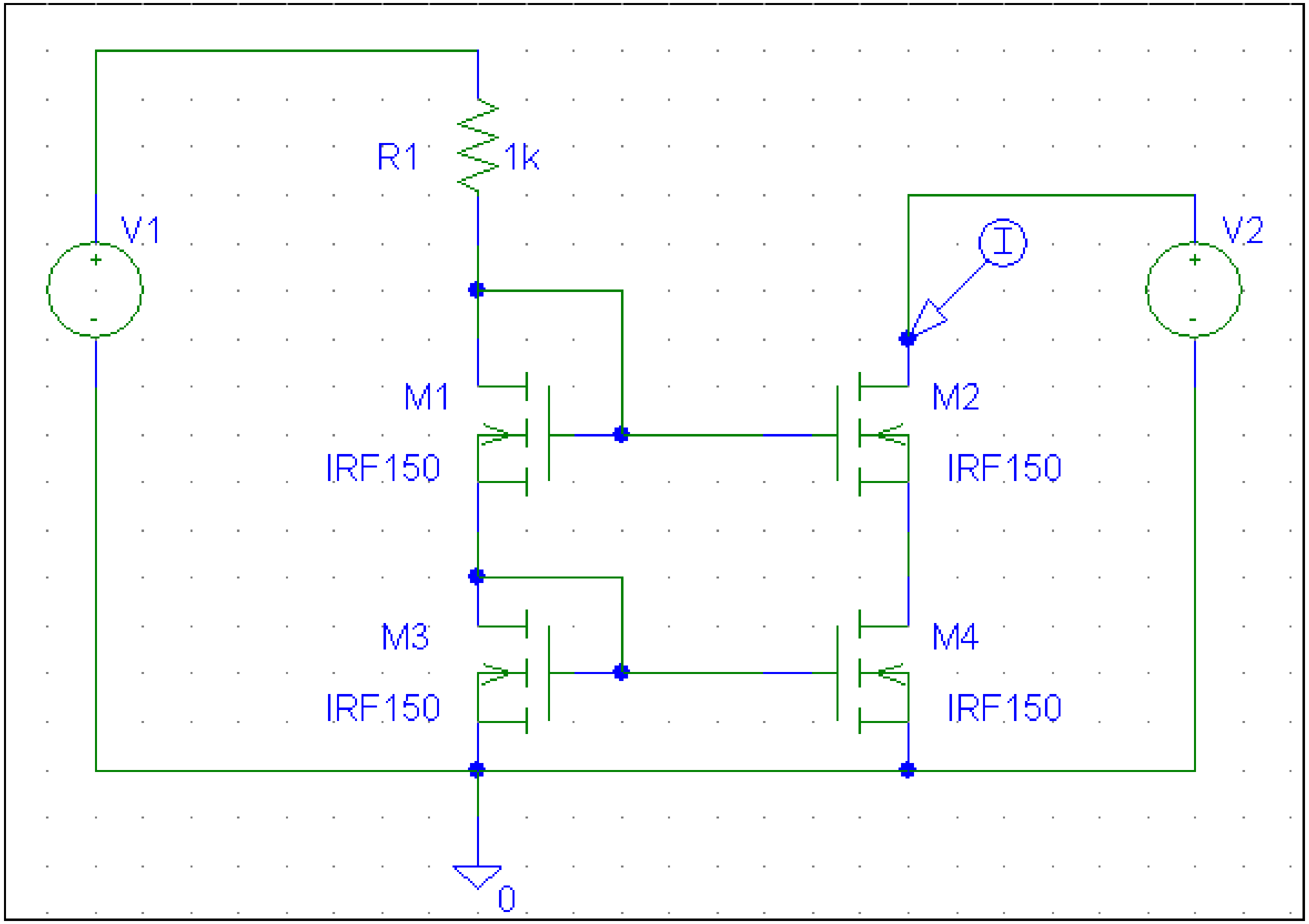
SIM 2.7: I_{D2} (V2)



SIMULATIONS for CMOS cascode current mirror

Output characteristic

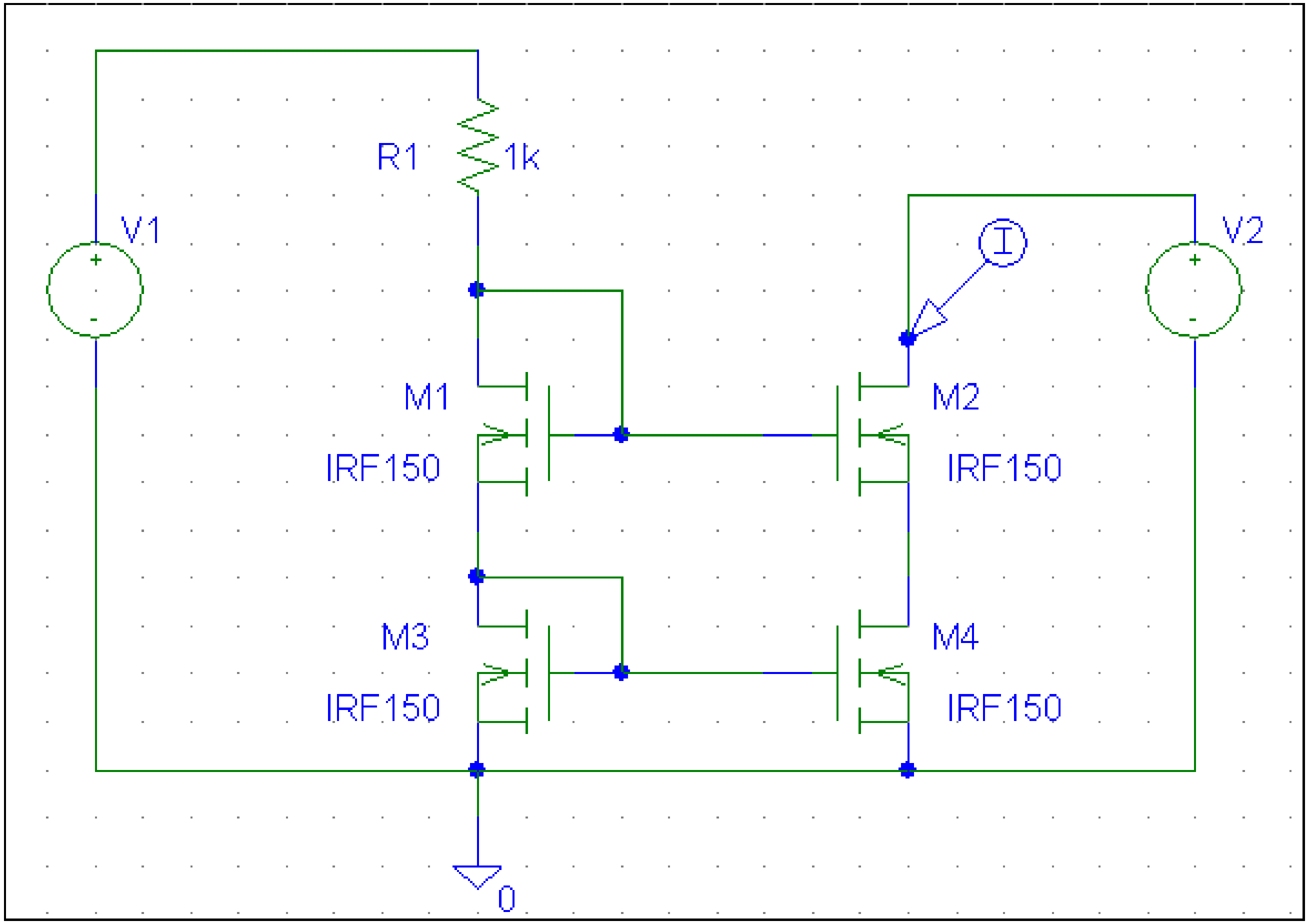
SIM 2.8: I_{D2} (V2), r_{ds2}, r_{ds4} - parameters



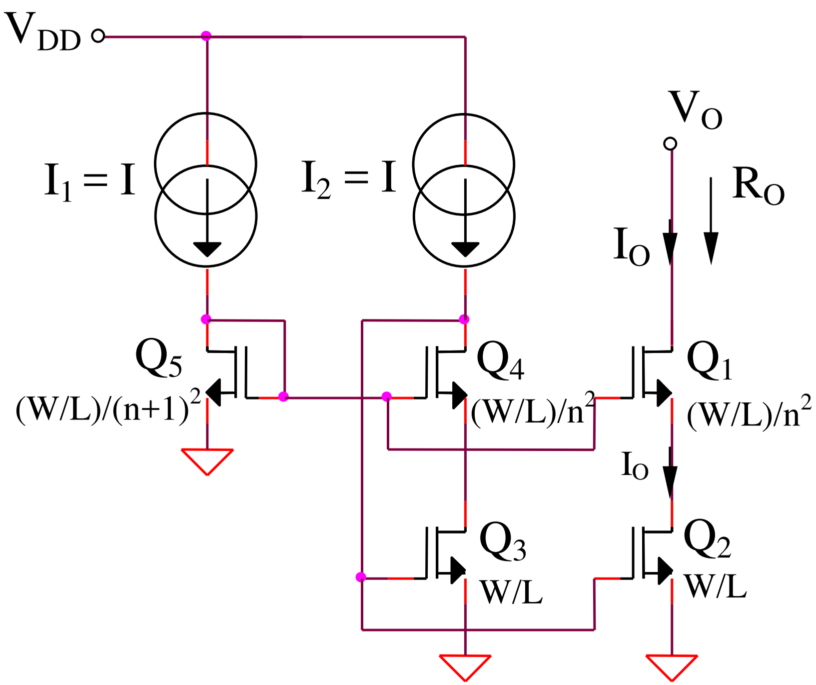
SIMULATIONS for CMOS cascode current mirror
Dependence of the output current on the supply voltage

SIMULATIONS for CMOS cascode current mirror
Dependence of the output current on the supply voltage

SIM 2.9: I_{D2} (V1)



MOS cascode current source (3)



Output current

$$I_O = I$$

Output resistance

$$R_0 = r_{ds1}\left(1 + g_{m1} r_{ds2}\right) \cong g_m r_{ds}^2$$

Minimum output voltage

$$\left.\begin{aligned} I &= \frac{K'}{2}\frac{W/L}{(n+1)^2}\left(V_{GS5} - V_T\right)^2 \\ I &= \frac{K'}{2}\frac{W/L}{n^2}\left(V_{GS1(4)} - V_T\right)^2 \\ I &= \frac{K'}{2}(W/L)\left(V_{GS2(3)} - V_T\right)^2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} V_{GS5} - V_T = (n+1)(V_{GS2(3)} - V_T) \\ V_{GS1(4)} - V_T = n(V_{GS2(3)} - V_T) \end{cases}$$

The drain-source voltage for Q_2 is:

$$V_{DS2} = V_{GS5} - V_{GS1} = (V_{GS5} - V_T) - (V_{GS1} - V_T) = V_{GS2} - V_T = V_{DS2sat}$$

So, T_2 is biased at the saturation limit and it results:

$$V_{O\min} = V_{DS1sat} + V_{DS2} = (n+1)(V_{GS2} - V_T) = (n+1)\sqrt{\frac{2I}{K}}$$

2.1.4. Self-biased current sources

2.1.4. Self-biased current sources

Current mirror

$$I_O = \frac{V_{CC} - v_{BE}}{R}$$

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} \cong 1$$

Widlar current source

$$I_O = \frac{V_{th}}{R_2}\ln\frac{I}{I_O}$$

$$\frac{\partial I_O}{\partial V_{CC}} = \frac{V_{th}}{R_2}\frac{I_O}{I}\left(\frac{1}{I_O}\frac{\partial I}{\partial V_{CC}} - \frac{I}{I_O^2}\frac{\partial I_O}{\partial V_{CC}}\right)$$

$$\frac{\partial I_O}{\partial V_{CC}} = \frac{\frac{V_{th}}{IR_2}}{1+\frac{V_{th}}{R_2 I_O}}\frac{\partial I}{\partial V_{CC}}$$

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} = \frac{V_{CC}}{I}\frac{1}{1+\frac{R_2 I_O}{V_{th}}}\frac{\partial I}{\partial V_{CC}} \cong \frac{1}{1+\frac{R_2 I_O}{V_{th}}}$$

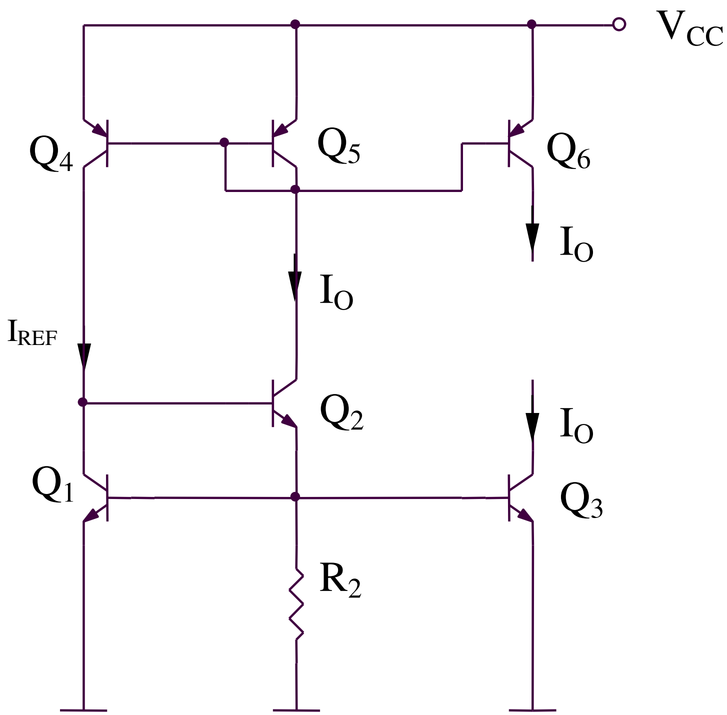
Current source using as reference the base-emitter voltage

$$I_O = \frac{V_{th}}{R_2} \ln \frac{V_{CC} - 2v_{BE}}{R_1 I_S}$$

$$\frac{\partial I_O}{\partial V_{CC}} \cong \frac{V_{th}}{R_2} \frac{R_1 I_S}{V_{CC} - 2v_{BE}} \frac{1}{R_1 I_S}$$

$$S_{V_{CC}}^{I_O} \cong \frac{V_{th}}{v_{BE}} \cong 4\%$$

Self-biased current source using as reference the base-emitter voltage



$$\left.\begin{array}{l} I_O = \dfrac{v_{BE1}}{R_2} = \dfrac{V_{th}}{R_2}\ln\dfrac{I_{REF}}{I_S} \\[2ex] \dfrac{I_{REF}}{I_O} = \dfrac{1+\dfrac{V_{CC}-2v_{BE}}{V_A}}{1+\dfrac{v_{BE}}{V_A}} \cong 1+\dfrac{V_{CC}-2v_{BE}}{V_A} \end{array}\right\} \Rightarrow$$

$$\Rightarrow I_O = \frac{V_{th}}{R_2}\ln\frac{I_O}{I_S} + \frac{V_{th}}{R_2}\ln\left(1+\frac{V_{CC}-2v_{BE}}{V_A}\right)$$

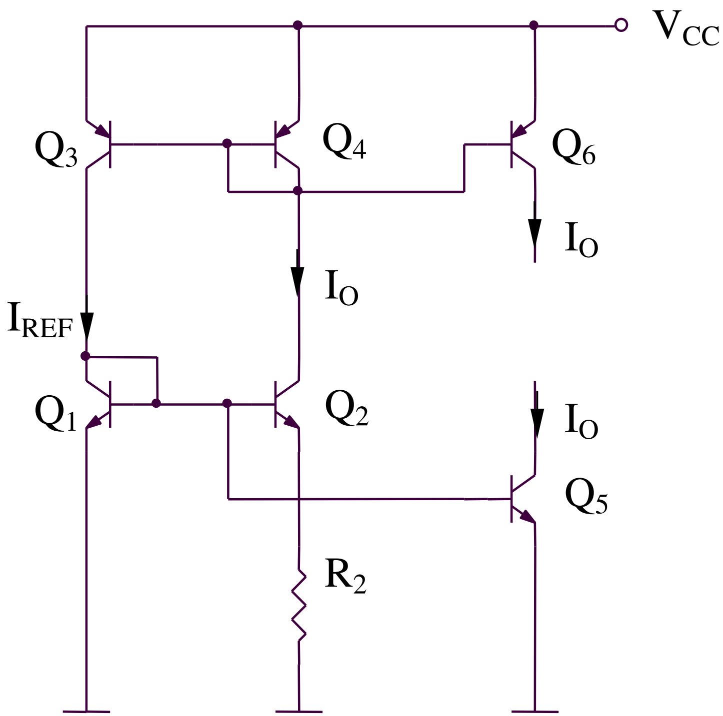
Deriving:

$$\frac{\partial I_O}{\partial V_{CC}} = \frac{V_{th}}{R_2(V_A+V_{CC})}$$

it results:

$$S_{V_{CC}}^{I_O} \cong \frac{V_{th}}{v_{BE}}\frac{1}{1+\dfrac{V_A}{V_{CC}}}$$

Widlar self-biased current source



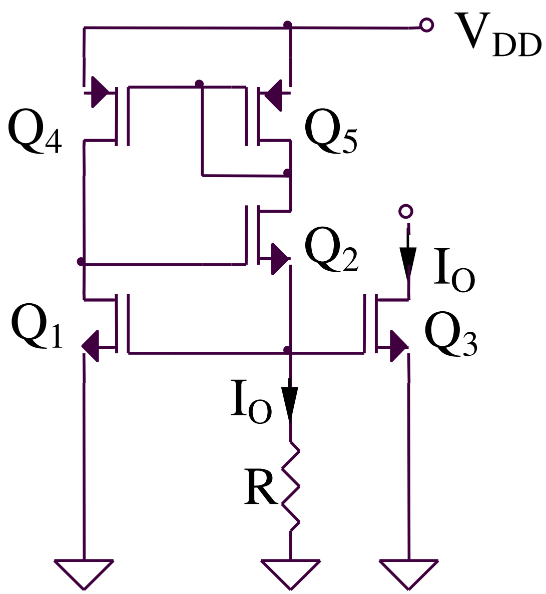
$$I_O = \frac{v_{BE1} - v_{BE2}}{R_2}$$

$$I_O = \frac{V_{th}}{R_2}\ln\left(\frac{I_{REF}}{I_O}\right) + \frac{V_{th}}{R_2}\ln\left(\frac{I_{S2}}{I_{S1}}\right)$$

$$I_O \cong \frac{V_{th}}{R_2}\ln\left(1 + \frac{V_{CC}}{V_A}\right) + \frac{V_{th}}{R_2}\ln\left(\frac{I_{S2}}{I_{S1}}\right)$$

$$S_{V_{CC}}^{I_O} \cong \frac{V_{CC}}{V_A}\frac{1}{\ln\left(\frac{I_{S2}}{I_{S1}}\right)}$$

Self-biased MOS current source (1)



Output current

$$I_O = \frac{V_{GS}}{R} = \frac{K}{2}\left(V_{GS} - V_T\right)^2$$

$$\frac{KR}{2}V_{GS}^2 - \left(1 + KRV_T\right)V_{GS} + \frac{KR}{2}V_T^2 = 0$$

Solving the equation in V_{GS} it results:

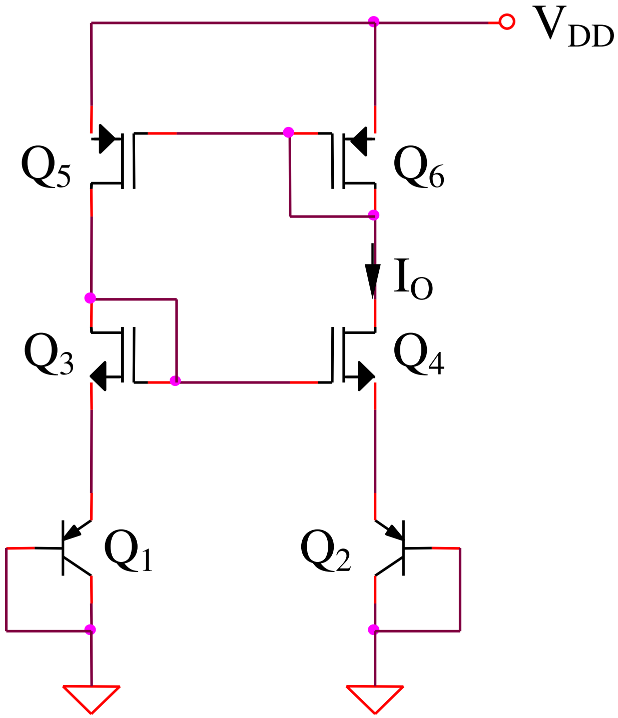
$$V_{GS1,2} = V_T + \frac{1}{KR} \pm \frac{\sqrt{2KRV_T + 1}}{KR}$$

$$V_{GS} = V_T + \frac{1}{KR} + \frac{\sqrt{2KRV_T + 1}}{KR}$$

So:

$$I_O = \frac{1}{KR^2}\left(1 + KRV_T + \sqrt{1 + 2KRV_T}\right)$$

Self-biased MOS current source (2)



Output current

$$V_T + \sqrt{\frac{2I_O}{4K}} + V_{th}\, ln\left(\frac{I_O}{I_S}\right) =$$

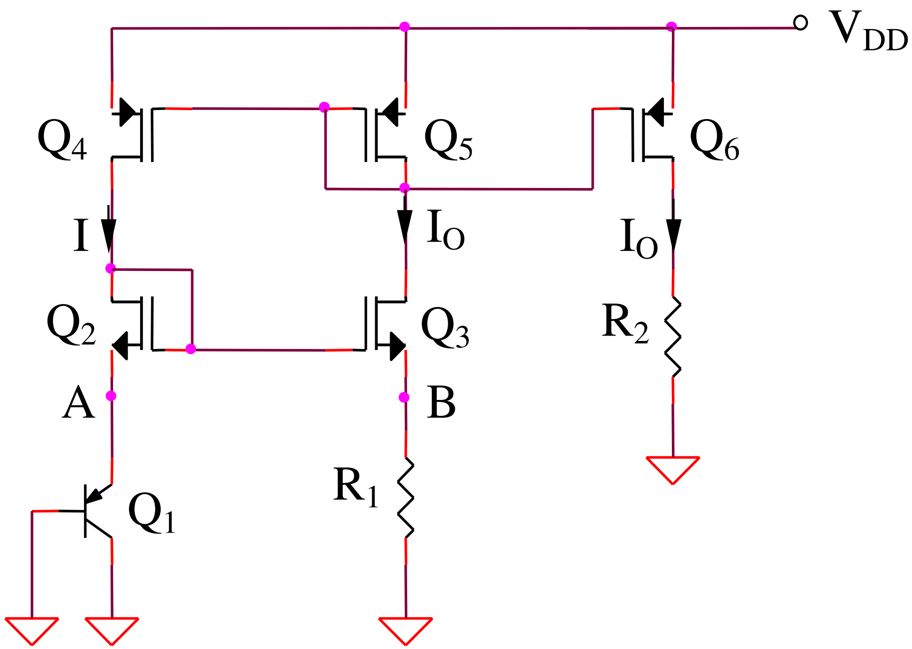
$$= V_T + \sqrt{\frac{2I_O}{K}} + V_{th}\, ln\left(\frac{I_O}{10 I_S}\right)$$

It results:

$$I_O = 2K\left[V_{th}\, ln(10)\right]^2$$

$$V_{th} = \frac{kT}{q}$$ - thermal voltage

Self-biased MOS current source (3)



Output current

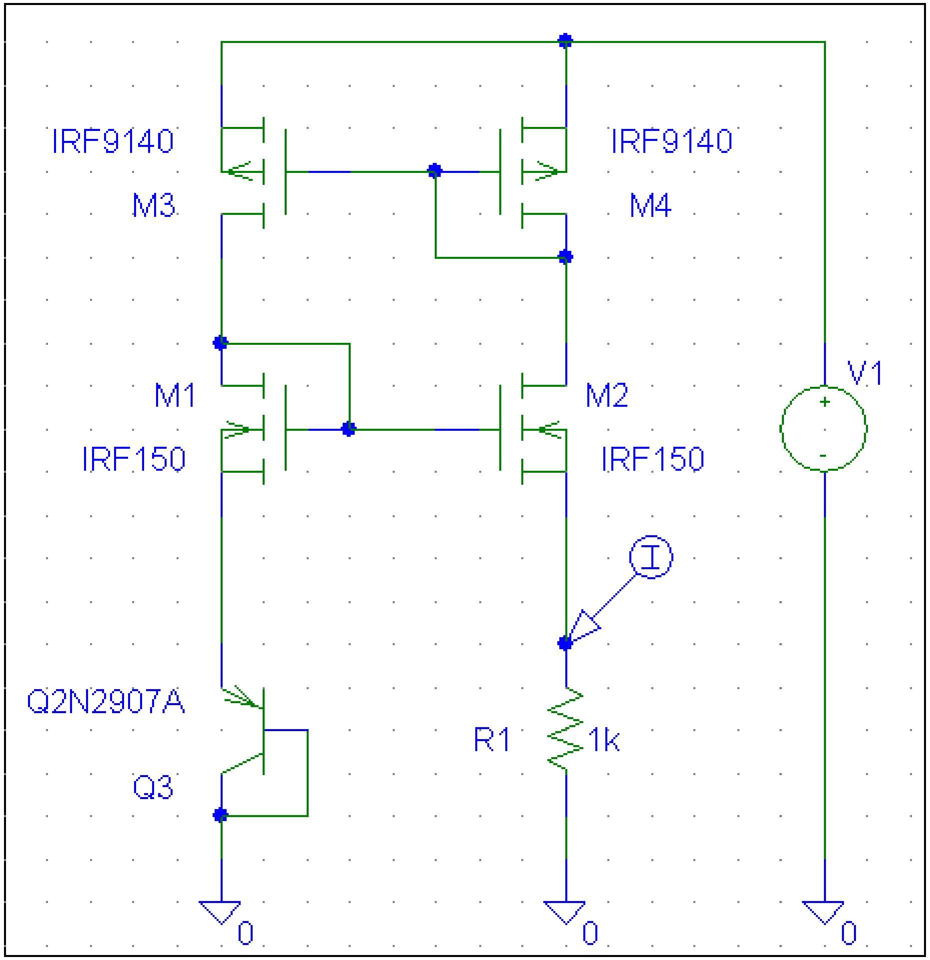
For identical MOS transistors, $V_A = V_B$, so:

$$I_O = \frac{V_{EB1}}{R_1}$$

SIMULATIONS for CMOS self-biased current source (3)
Dependence of the output current on the supply voltage

SIMULATIONS for CMOS self-biased current source (3)
Dependence of the output current on the supply voltage

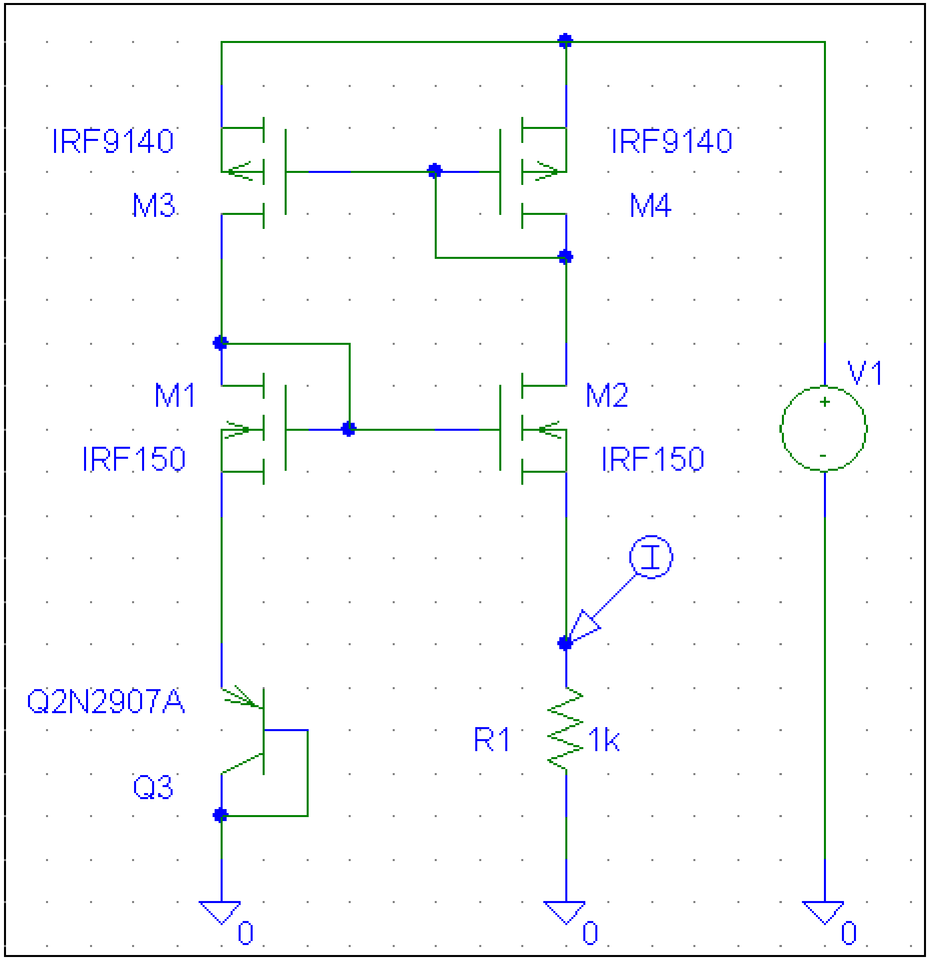
SIM 2.10: I_{D2} (V1)



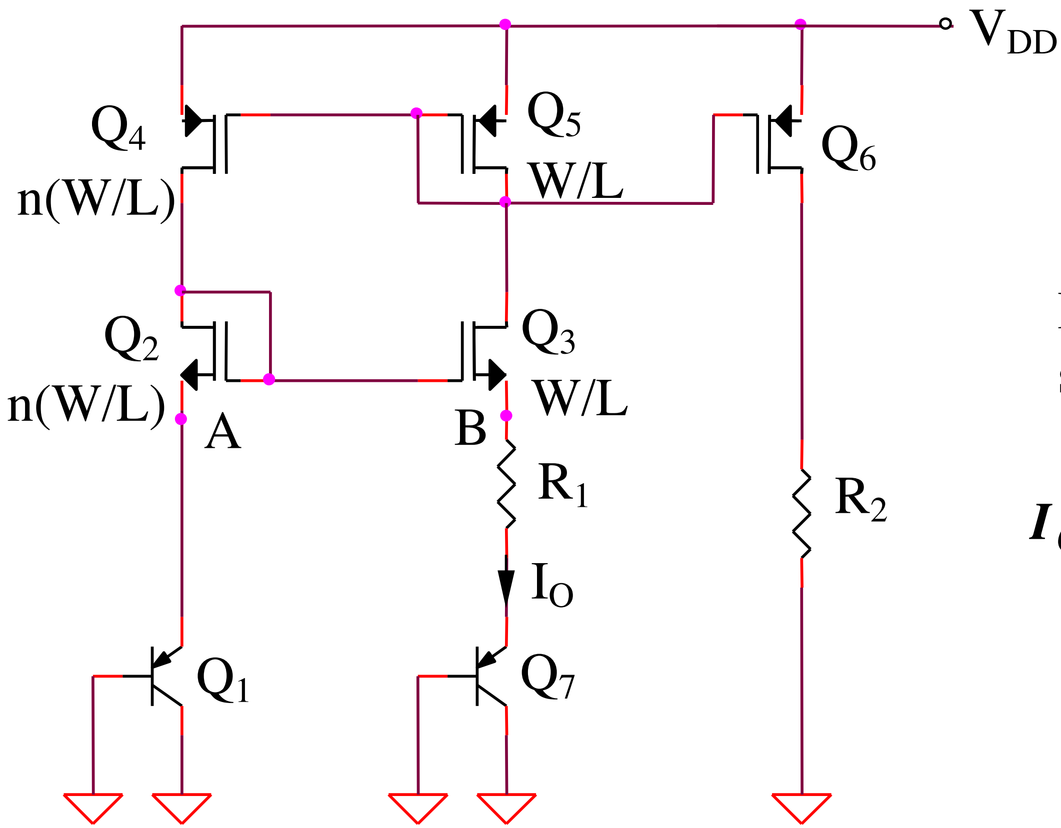
SIMULATIONS for CMOS self-biased current source (3)
Dependence of the output current on temperature

SIMULATIONS for CMOS self-biased current source (3)
Dependence of the output current on temperature

SIM 2.11: I_{D2} (t)



Self-biased MOS current source (4)



Output current

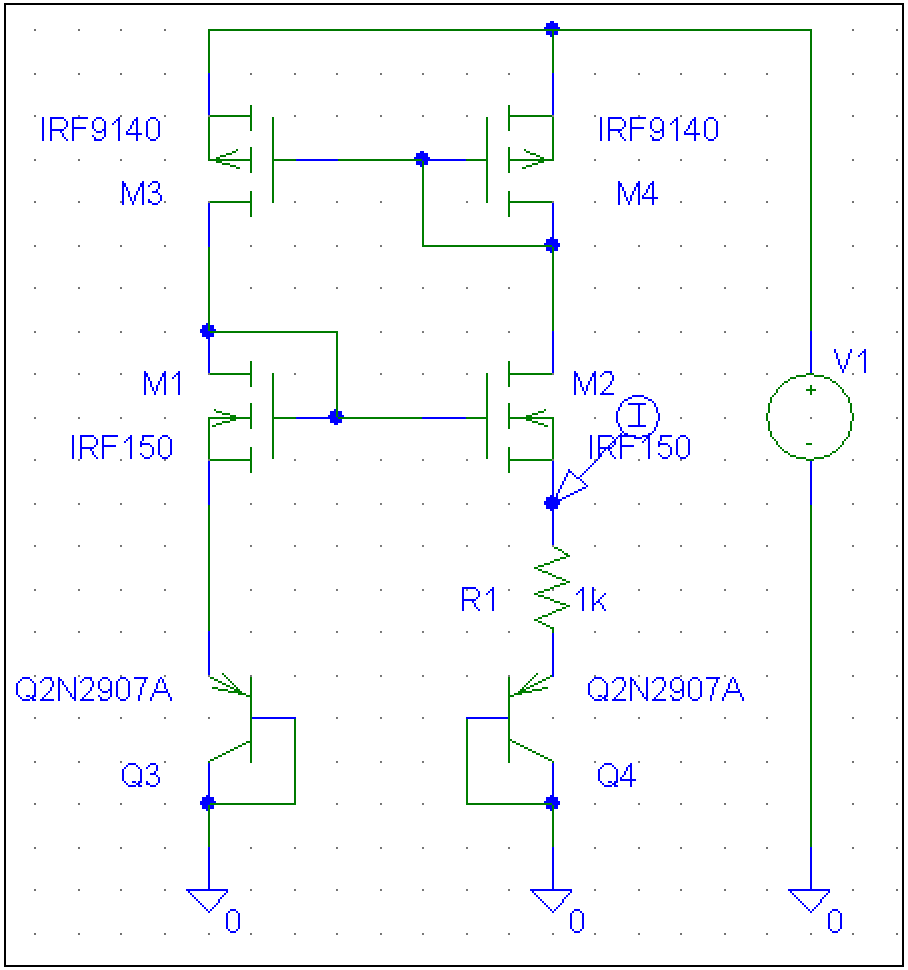
It can be demonstrated that $V_A = V_B$, so:

$$I_O = \frac{|V_{BE1}| - |V_{BE7}|}{R_1} = \frac{V_{th}}{R_1} ln(n)$$

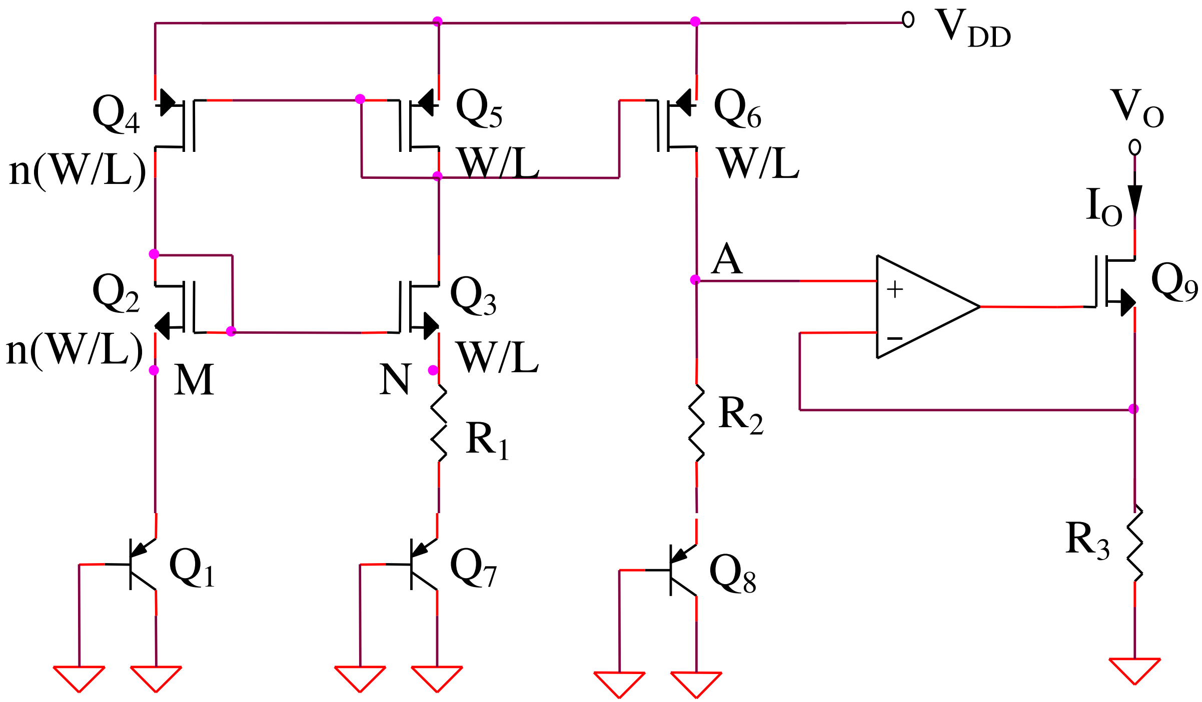
SIMULATIONS for CMOS self-biased current source (4)
Dependence of the output current on temperature

SIMULATIONS for CMOS self-biased current source (4)
Dependence of the output current on temperature

SIM 2.12: I_{D2} (t)



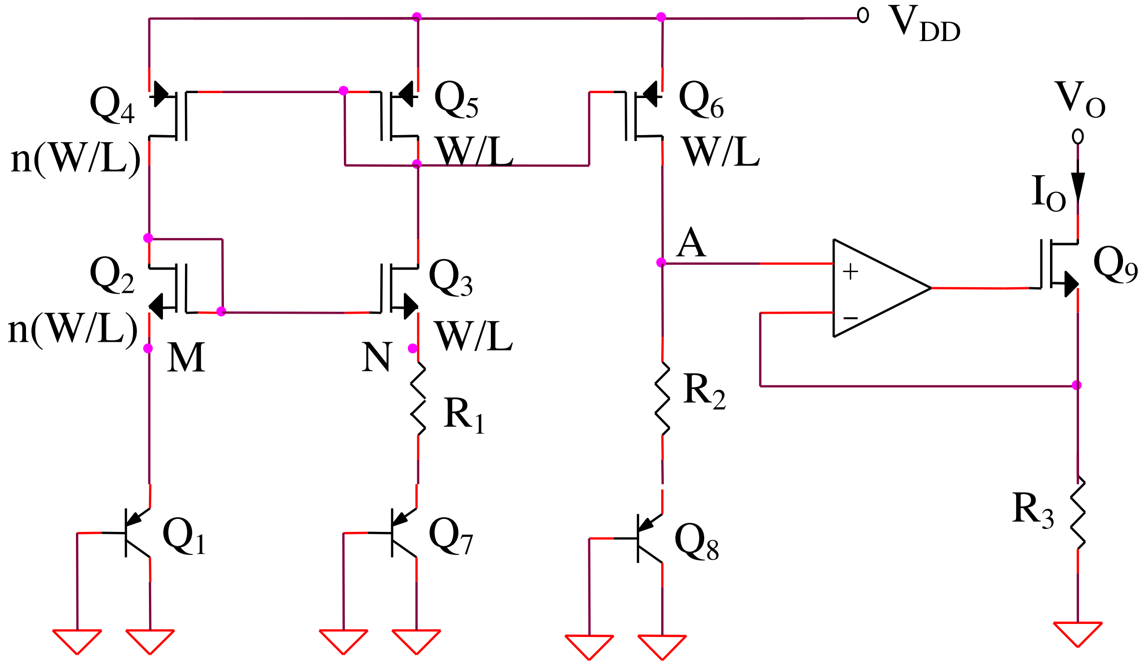
Self-biased MOS current source (5)



$$\left.\begin{aligned} V_{GS2} &= V_{GS3} \\ V_{R_2} = R_2 \frac{V_{EB1} - V_{EB7}}{R_1} &= \frac{R_2}{R_1} V_{th} \ln(n) \end{aligned}\right\} \Rightarrow I_O(T) = \frac{I}{R_3}\left[\frac{R_2}{R_1} V_{th} \ln(n) + V_{EB8}(T)\right]$$

$$V_{EB}(T) = A + BT + CT \ln\left(\frac{T}{T_O}\right)$$

Self-biased MOS current source (5) – cont.



$$\Rightarrow I_O(T) = \frac{I}{R_3}\left[\frac{R_2}{R_1}\frac{kT}{q}\ln(n) + A + BT + CT\ln\left(\frac{T}{T_O}\right)\right]$$

The condition of linear curvature correction can be written as follows:

$$B + \frac{R_2}{R_1}\frac{k}{q}\ln(n) = 0$$

It results:

$$I_O(T) = \frac{I}{R_3}\left[A + CT\ln\left(\frac{T}{T_O}\right)\right]$$

2.2. Reference voltage sources

2.2.1. Classification

2.2. Reference voltage sources

2.2.1. Classification

I. Elementary voltage sources
- reduced complexity
- poor performances

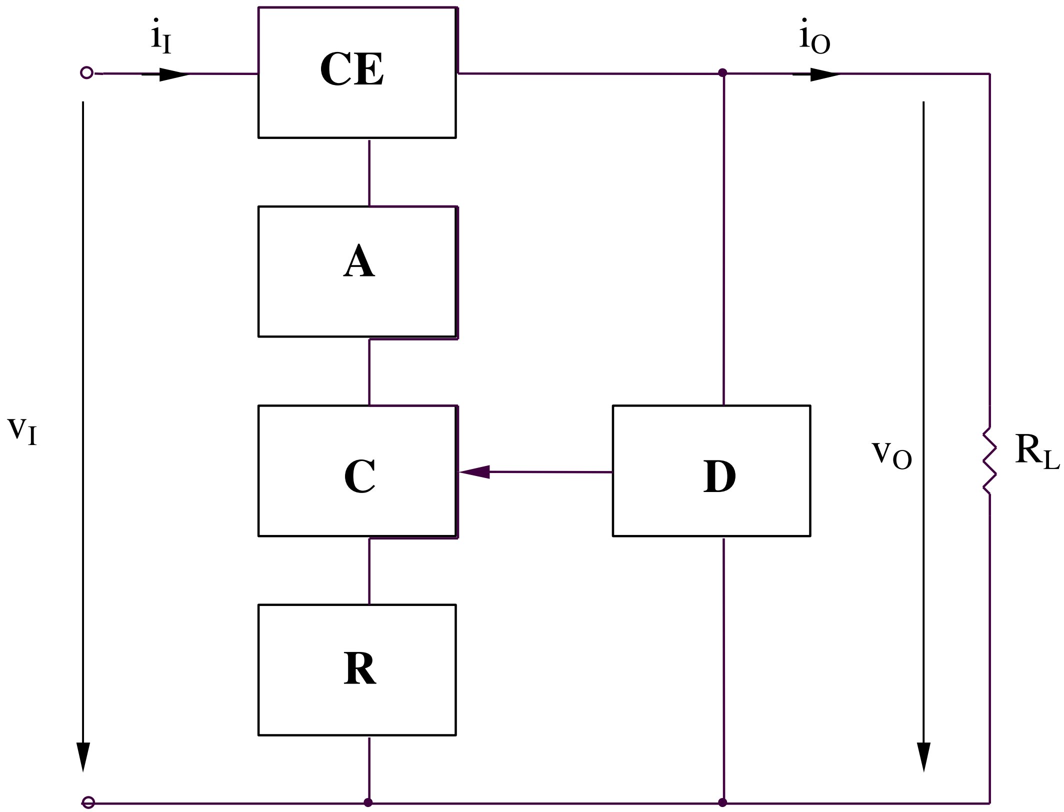
II. Voltage sources with reaction
- reduced output resistance
- increased complexity

III. Temperature-compensated voltage sources
- reduced dependence on temperature of the output voltage
- increased complexity

2.2.2. Voltage sources with reaction

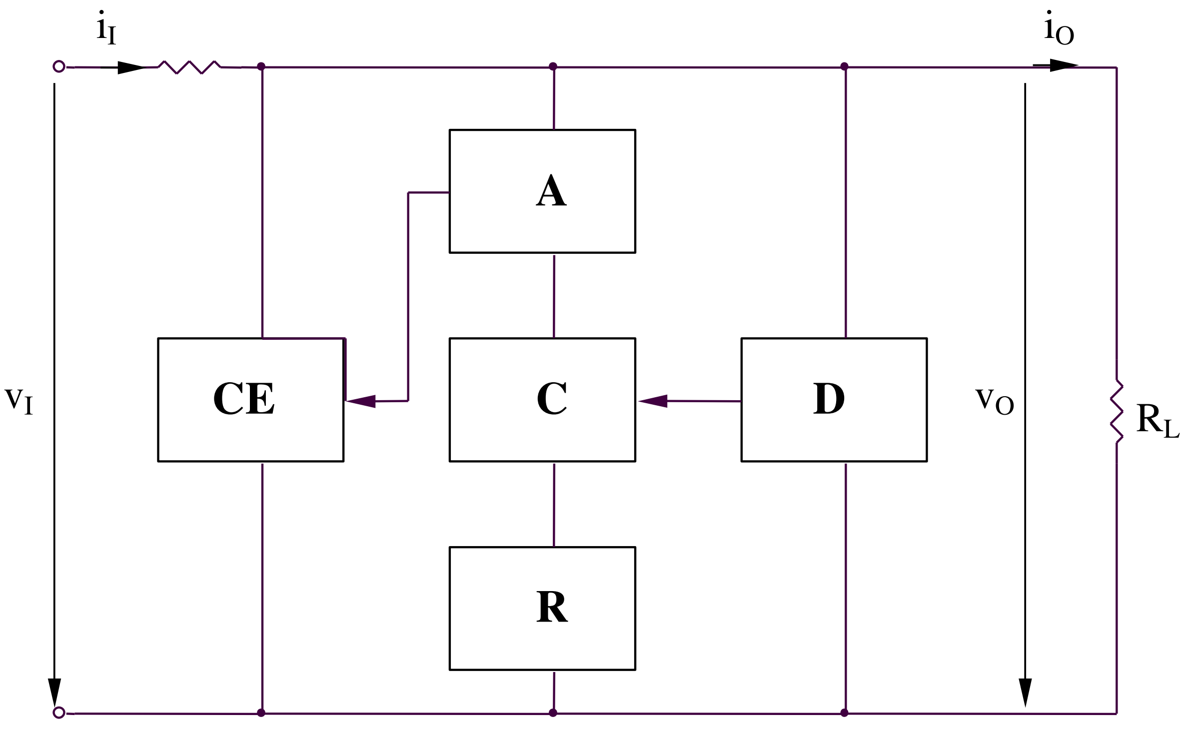
2.2.2. Voltage sources with reaction

Voltage sources with series regulation (block diagram)



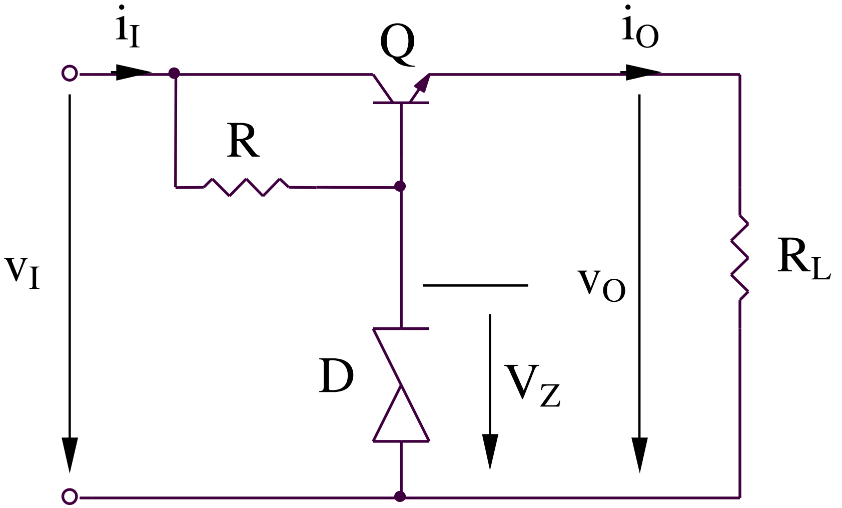
D = dividing circuit
C = comparison circuit
R = reference circuit
A = amplifier
CE = control element

Voltage sources with parallel regulation (block diagram)

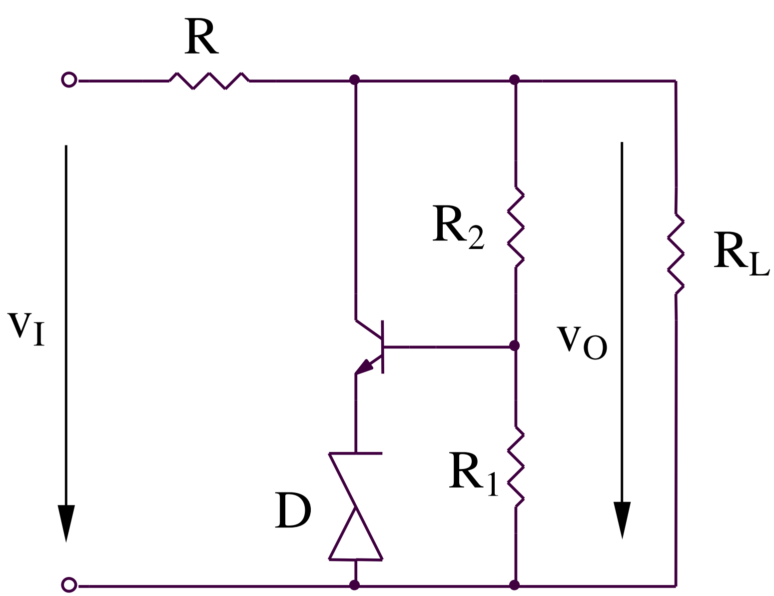


D = dividing circuit
C = comparison circuit
R = reference circuit
A = amplifier
CE = control element

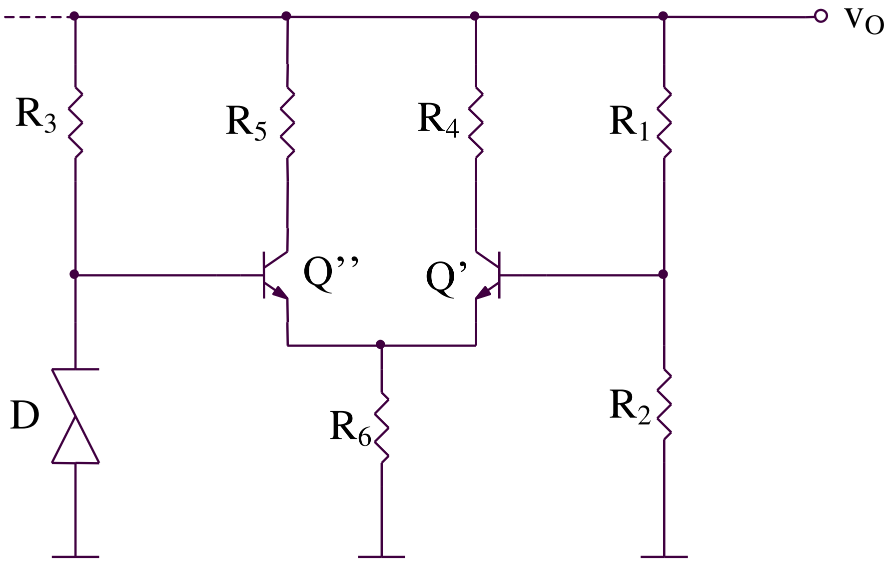
Examples of voltage sources

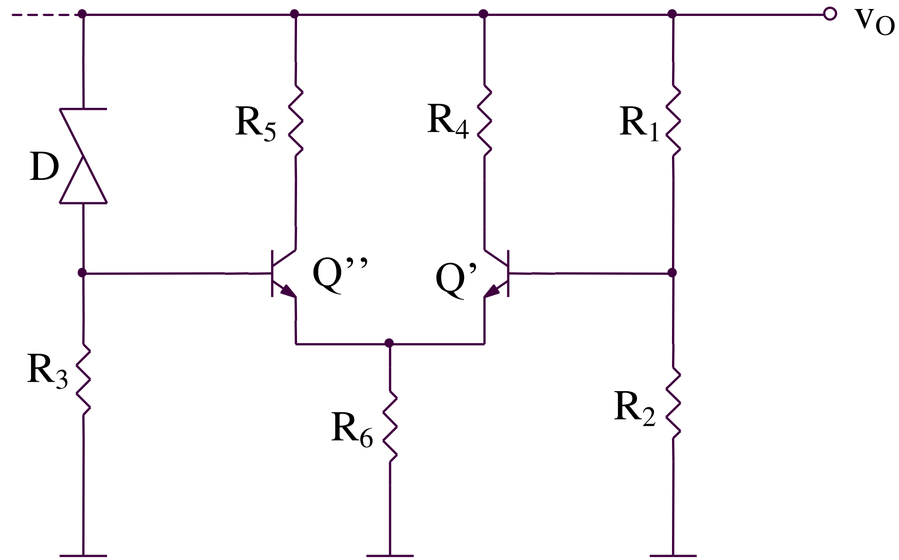


$$v_O = V_Z - V_{BE}$$



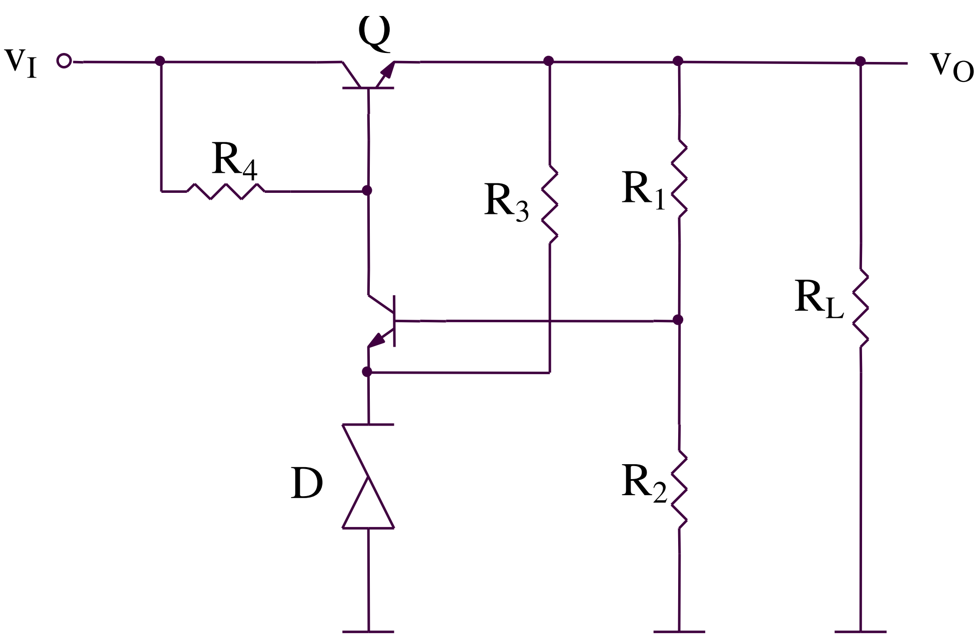
$$v_O = \left(V_{BE} + V_Z\right)\left(1 + \frac{R_2}{R_1}\right) > V_Z$$



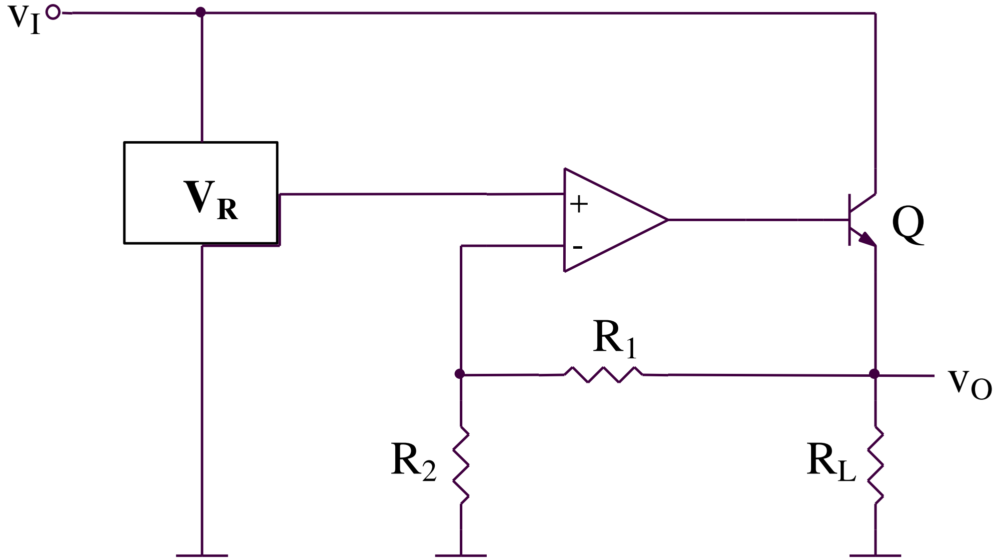


$$v_O = V_Z\left(1+\frac{R_1}{R_2}\right)$$

$$v_O = V_Z\left(1+\frac{R_2}{R_1}\right)$$

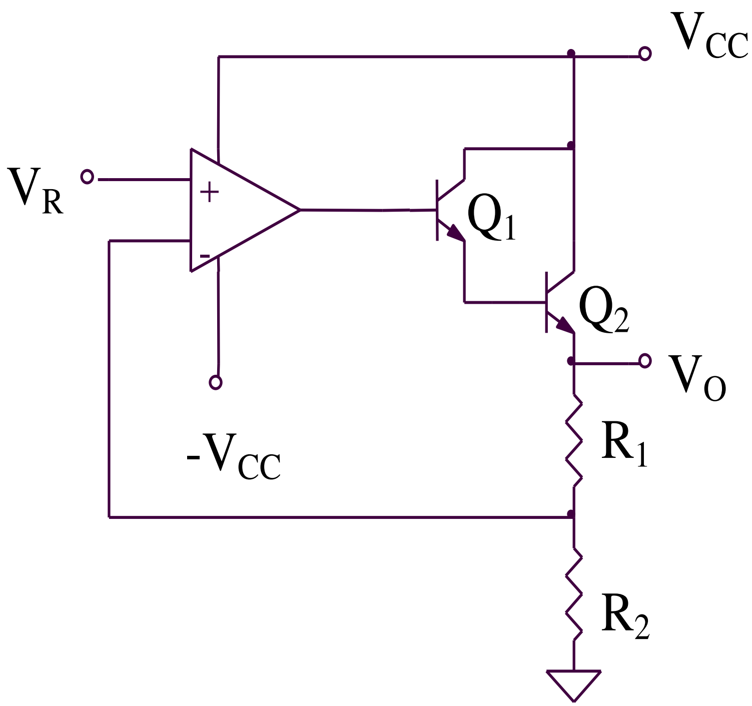


$$v_O = \left(V_Z + V_{BE}'\right)\left(1+\frac{R_1}{R_2}\right)$$



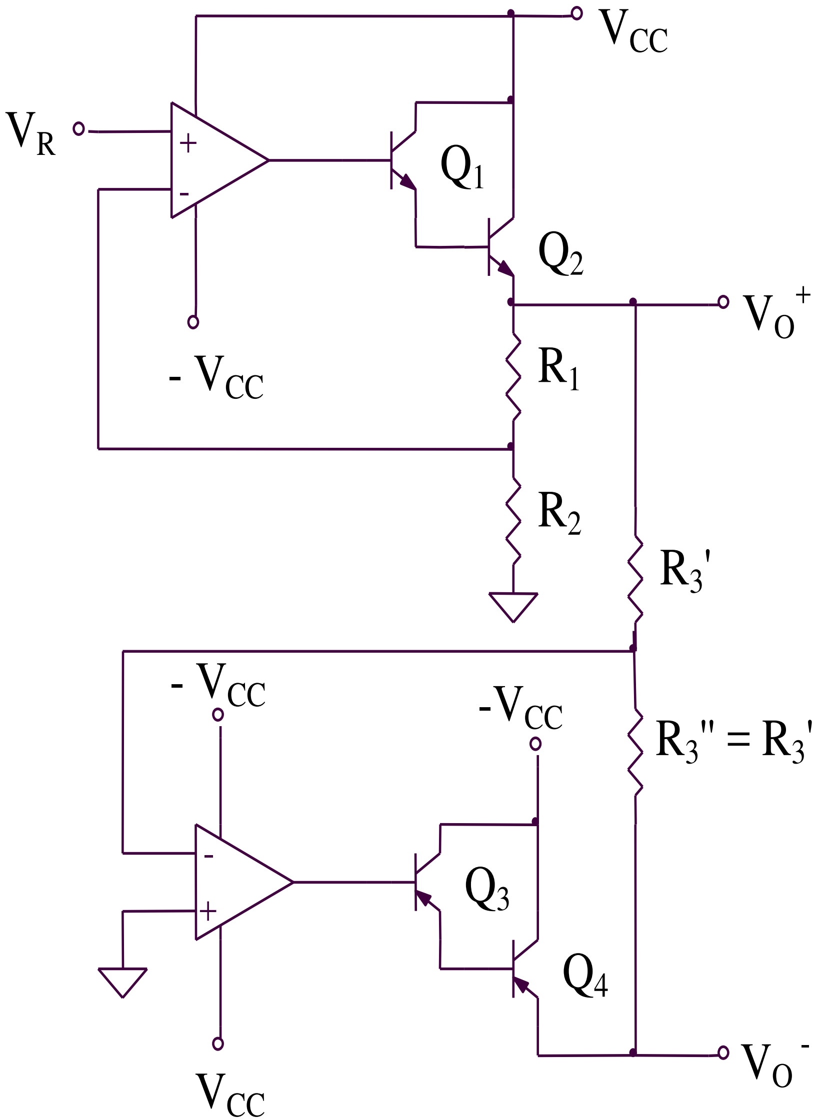
$$V_R = V_O \frac{R_2}{R_1 + R_2}$$

$$V_O = V_R\left(1 + \frac{R_1}{R_2}\right)$$



$$V_R = V_O \frac{R_2}{R_1 + R_2}$$

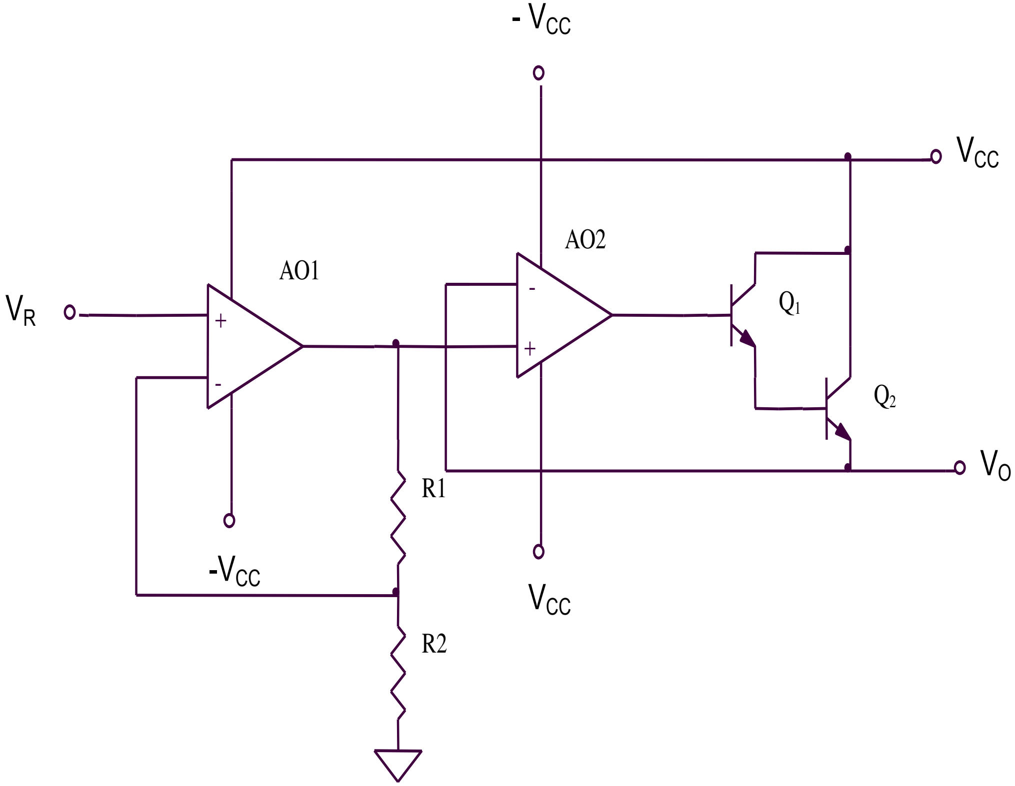
$$V_O = V_R\left(1 + \frac{R_1}{R_2}\right)$$



$$V_R = V_O^{+} \frac{R_2}{R_1 + R_2}$$

$$V_O^{+} = V_R\left(1 + \frac{R_1}{R_2}\right)$$

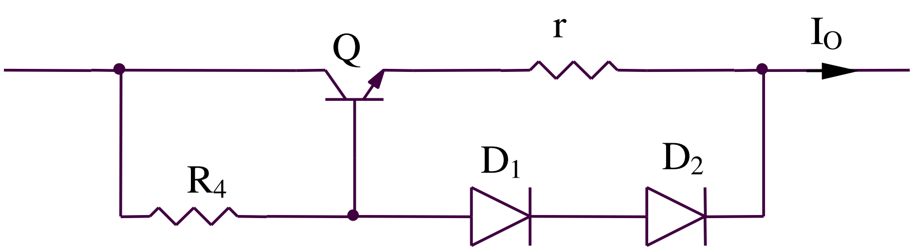
$$\frac{V_O^{+}}{R_3'} = -\frac{V_O^{-}}{R_3''} \Rightarrow V_O^{-} = -V_O^{+}$$



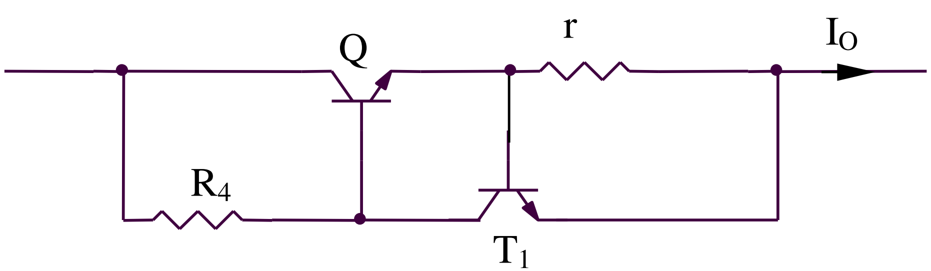
$$V_R = V_O \frac{R_2}{R_1 + R_2}$$

$$V_O = V_R \left(1 + \frac{R_1}{R_2} \right)$$

Overload protection (1)

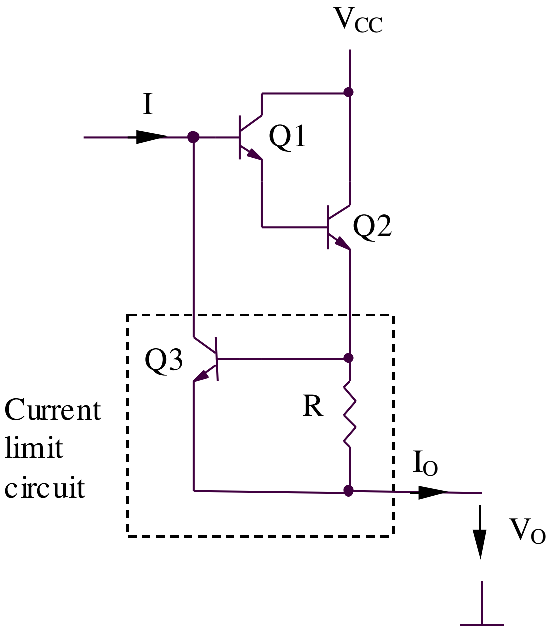


$$I_{OL} = \frac{V_{D1} + V_{D2} - V_{BE}}{r} \cong \frac{V_D}{r}$$



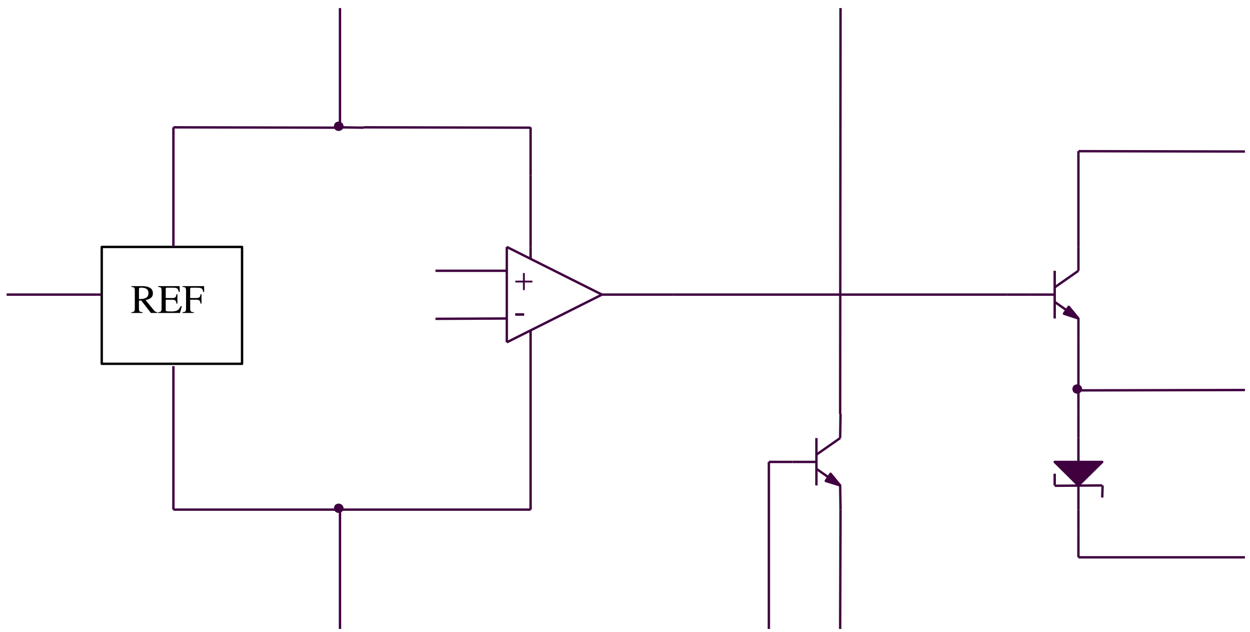
$$I_{OL} = \frac{V_{BE}}{r}$$

Overload protection (2)

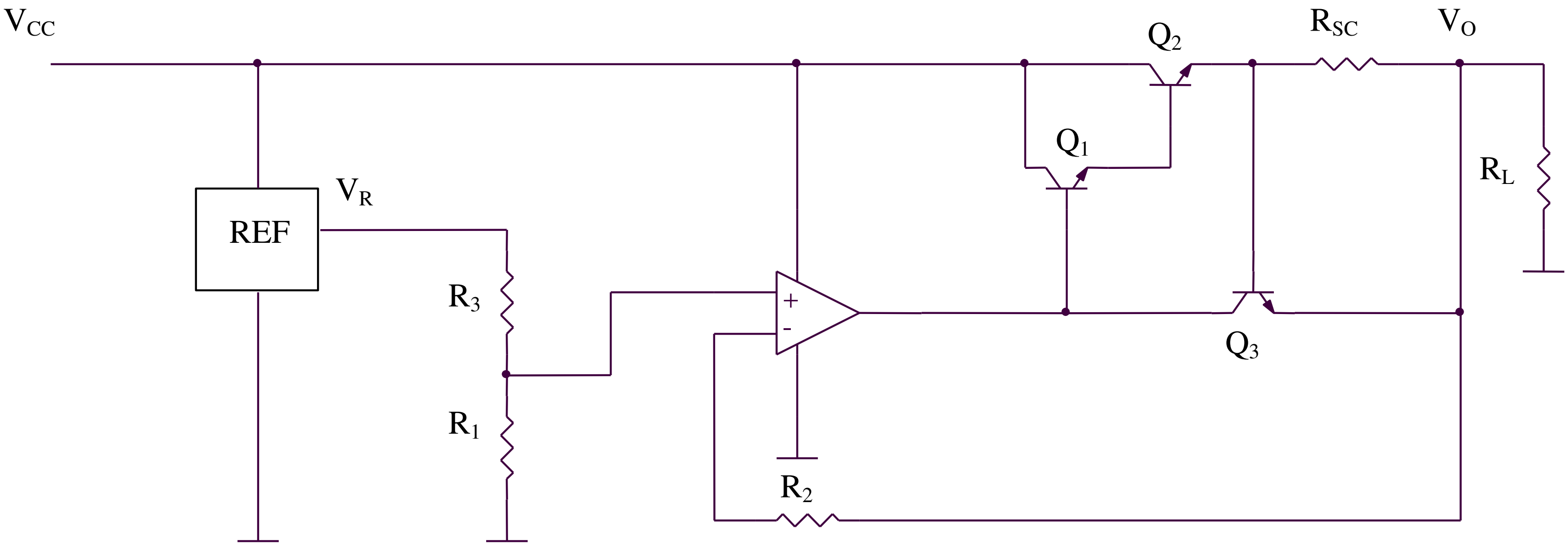


$$I_{OL} = \frac{V_{BE3}}{R}$$

BA 723 circuit



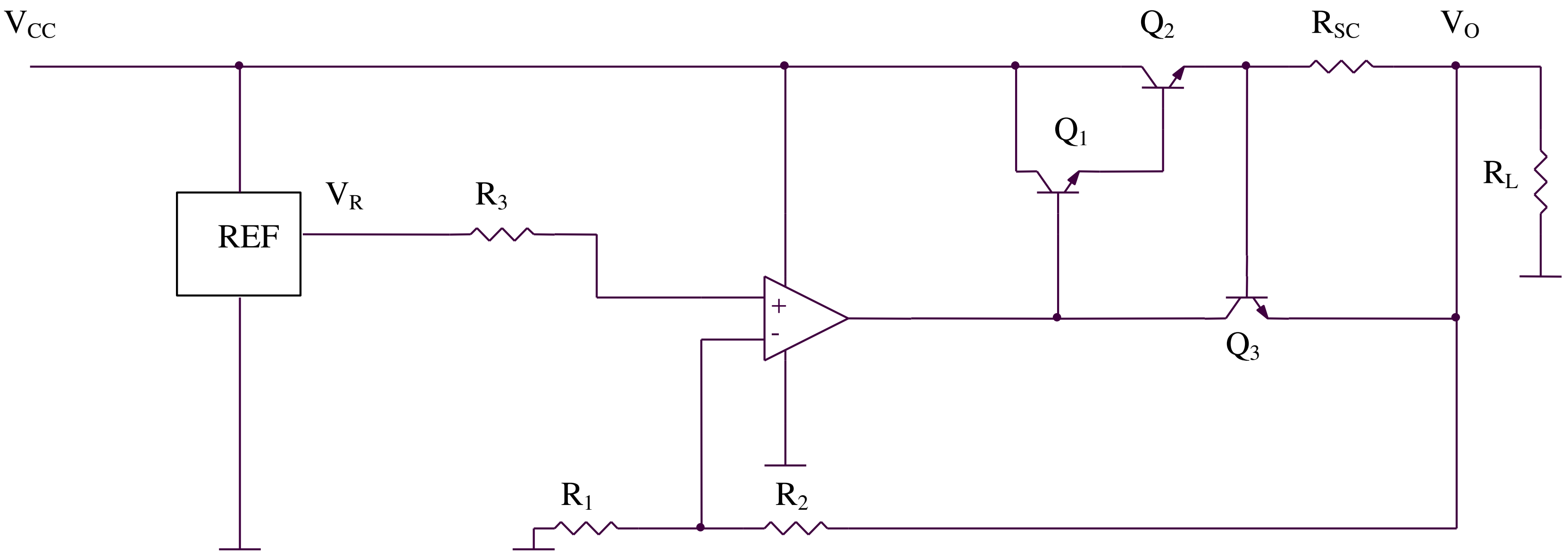
Application for $V_O < V_R$



$$V_O = V_R \frac{R_1}{R_1 + R_3} < V_R$$

$$I_{Osc} = \frac{V_{BE}}{R_{sc}}$$

Application for $V_O > V_R$



$$V_O \frac{R_1}{R_1 + R_2} = V_R \Rightarrow V_O = V_R\left(1 + \frac{R_2}{R_1}\right) > V_R$$

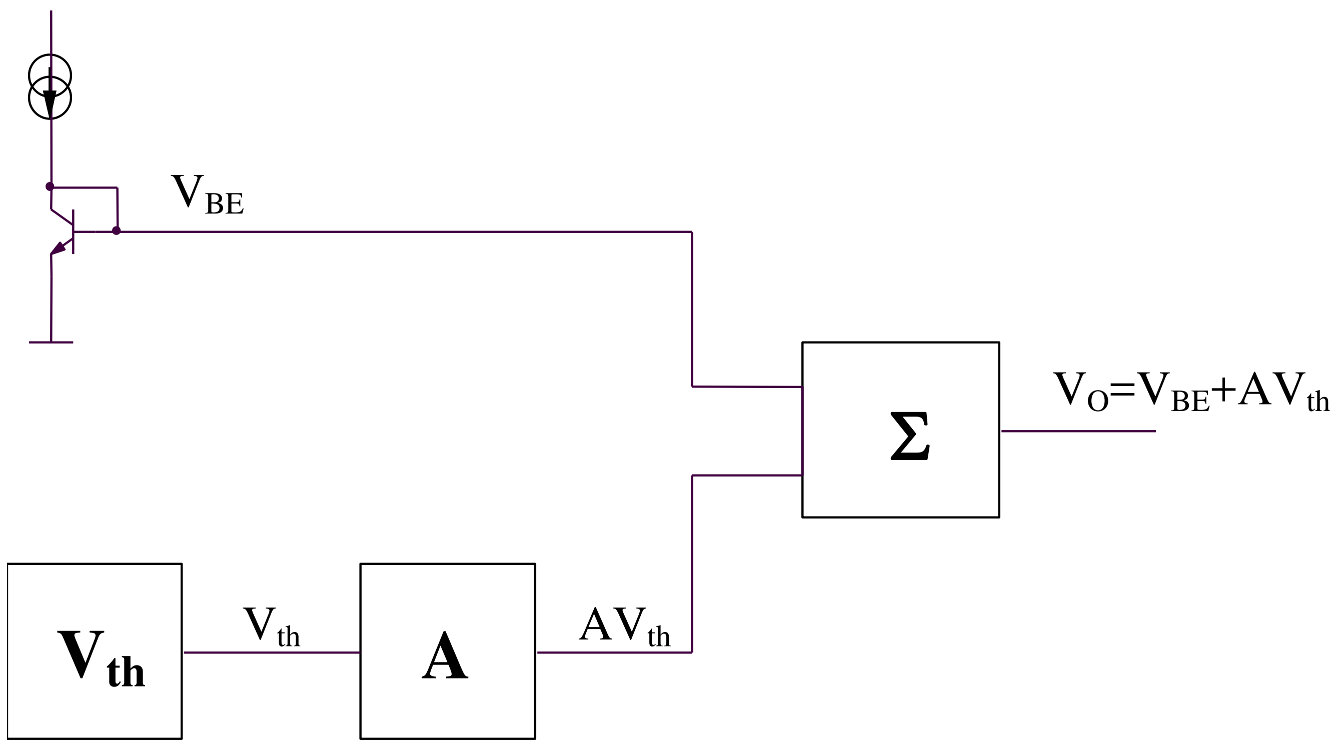
$$I_{Osc} = \frac{V_{BE}}{R_{sc}}$$

2.2.3. Temperature-compensated voltage sources

2.2.3. Temperature-compensated voltage sources

Bandgap voltage references

Are based on the compensation of opposite temperature dependencies of base-emitter voltage and thermal voltage $V_{th} = kT/q$. It is possible to obtain a null temperature coefficient by considering a weight sum of these terms.



The temperature dependence of V_{BE}

$$\left.\begin{aligned} V_{BE}(T) &= V_{th}\ln\left[\frac{I_C(T)}{I_S(T)}\right] \\ I_S(T) &= CT^{\eta}\exp\left(-\frac{E_{GO}}{V_{th}}\right) \end{aligned}\right\} \Rightarrow V_{BE}(T) = E_{G0} + \frac{kT}{q}\ln\left[\frac{I_C(T)}{CT^{\eta}}\right]$$

$$\left.\begin{aligned} V_{BE}(T) &= E_{G0} + \frac{kT}{q}\ln\left[\frac{I_C(T)}{CT^{\eta}}\right] \\ V_{BE}(T_O) &= E_{GO} + \frac{kT_O}{q}\ln\left[\frac{I_C(T_O)}{CT_O^{\eta}}\right] \\ I_C(T) &= BT^{\alpha} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow V_{REF}(T) = E_{GO} + \frac{V_{BE}(T_O) - E_{GO}}{T_O}T + (\alpha - \eta)\frac{KT}{q}\ln\left(\frac{T}{T_O}\right)$$

$$\frac{V_{BE}(T_O) - E_{GO}}{T_O} \cong -2.1mV/K < 0$$

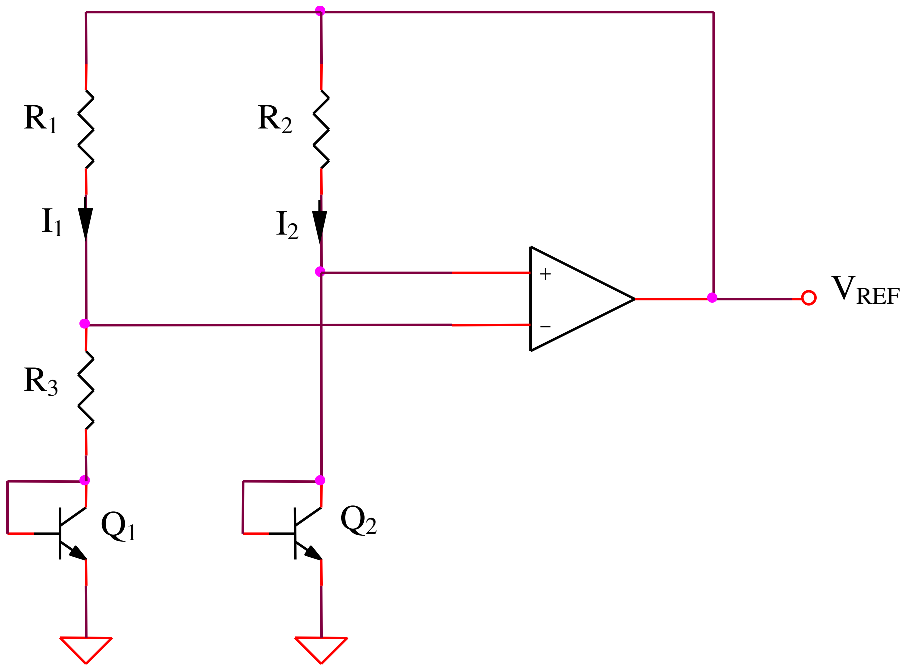
The operation of voltage reference

$$\left.\begin{aligned} V_{REF}(T) &= DV_{th} + V_{BE2}(T) \\ V_{BE}(T) &= A + BT + CT\ln\left(\frac{T}{T_O}\right) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow V_{REF}(T) = A + \left(B + D\frac{k}{q}\right)T + CT\ln\left(\frac{T}{T_O}\right)$$

$$B + D\frac{k}{q} = 0 \Rightarrow V_{REF}(T) = A + CT\ln\left(\frac{T}{T_O}\right)$$

Example (1)



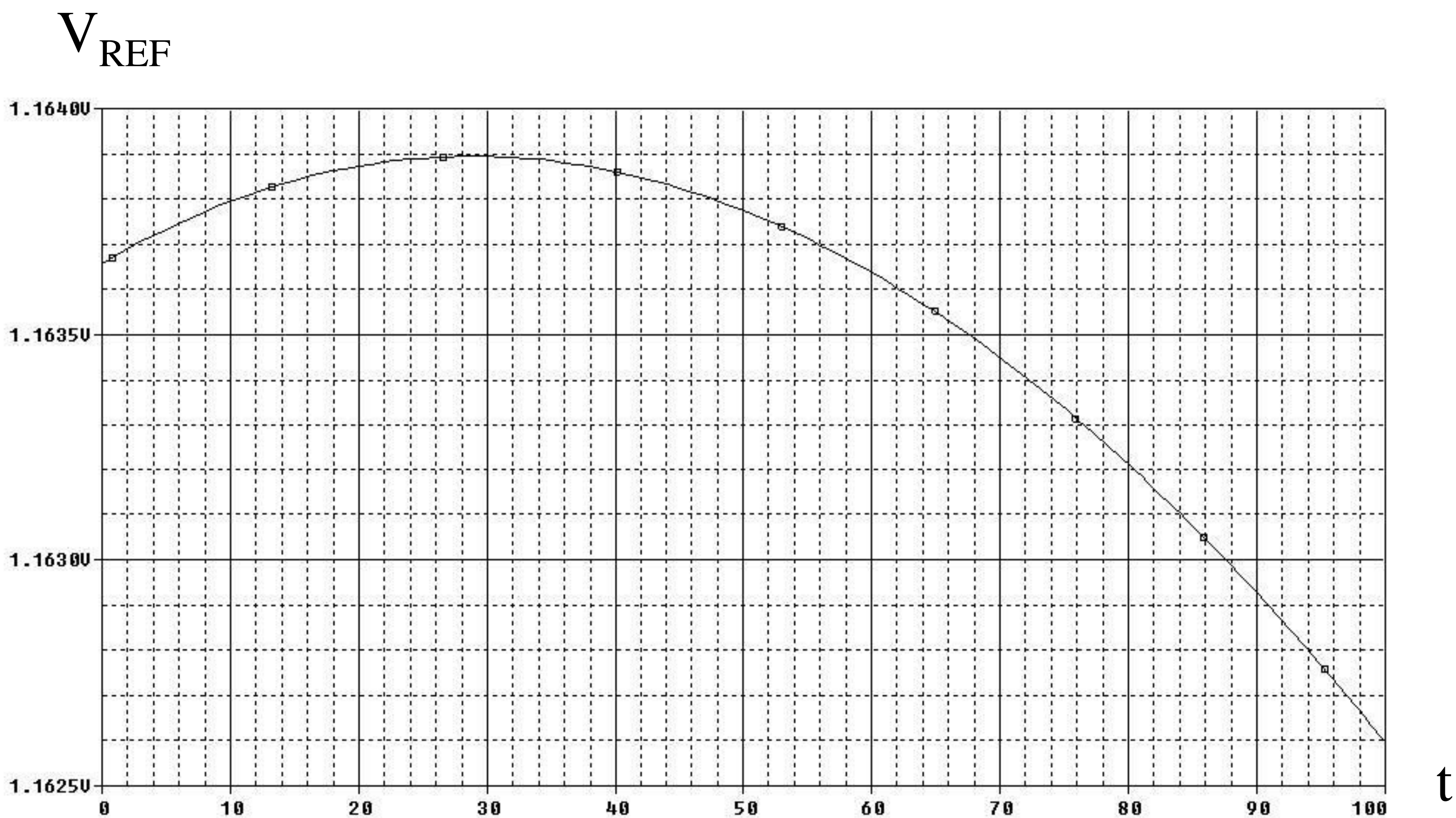
$$\left.\begin{aligned} I_1 = \frac{V_{BE2} - V_{BE1}}{R_3} = \frac{kT}{qR_3}\ln\left(\frac{I_2}{I_1}\right) \\ I_1 R_1 = I_2 R_2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \quad I_1 = \frac{kT}{qR_3}\ln\left(\frac{R_1}{R_2}\right)$$

$$\left.\begin{aligned} V_{REF}(T) = I_1(T)R_1 + V_{BE2}(T) \\ V_{BE}(T) = A + BT + CT\ln\left(\frac{T}{T_O}\right) \end{aligned}\right\} \Rightarrow$$

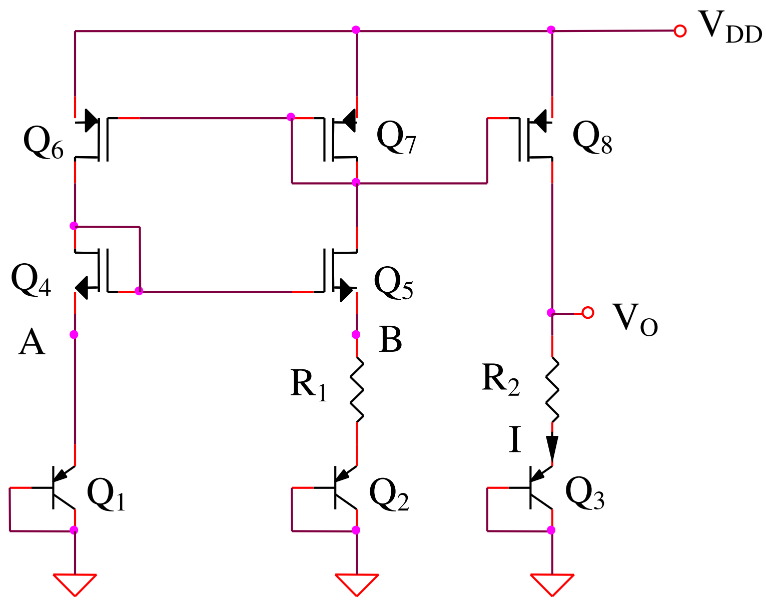
$$\Rightarrow \quad V_{REF}(T) = A + \left[B + \frac{k}{q}\frac{R_1}{R_3}\ln\left(\frac{R_1}{R_2}\right)\right]T + CT\ln\left(\frac{T}{T_O}\right)$$

$$B + \frac{k}{q}\frac{R_1}{R_3}\,ln\left(\frac{R_1}{R_2}\right) = 0 \Rightarrow V_{REF}(T) = A + CT\,ln\left(\frac{T}{T_O}\right) \cong A \cong 1.2V$$



V_REF
1.1640V
1.1635V
1.1630V
1.1625V
0
10
20
30
40
50
60
70
80
90
100
t

Example (2)



$$V_A - V_B = V_{GS5} - V_{GS4} = (V_{GS5} - V_T) - (V_{GS4} - V_T) = \sqrt{\frac{2I_{D5}}{K_5}} - \sqrt{\frac{2I_{D4}}{K_4}}$$

$$V_A - V_B = \sqrt{\frac{2I_{D5}}{K_5}}\left(1 - \sqrt{\frac{I_{D4}}{I_{D5}}\frac{K_5}{K_4}}\right) = \sqrt{\frac{2I_{D5}}{K_5}}\left(1 - \sqrt{\frac{I_{D6}}{I_{D7}}\frac{(W/L)_5}{(W/L)_4}}\right)$$

$$V_A - V_B = \sqrt{\frac{2I_{D5}}{K}}\left(1 - \sqrt{\frac{(W/L)_5}{(W/L)_4}\frac{(W/L)_6}{(W/L)_7}}\right)$$

For: $$\frac{(W/L)_4}{(W/L)_5} = \frac{(W/L)_6}{(W/L)_7} \Rightarrow V_A = V_B$$

$$\Rightarrow V_O(T) = |V_{BE_3}(T)| + I(T)R_2 = |V_{BE_3}(T)| + \frac{|V_{BE_1}(T)| - |V_{BE_2}(T)|}{R_1} R_2$$

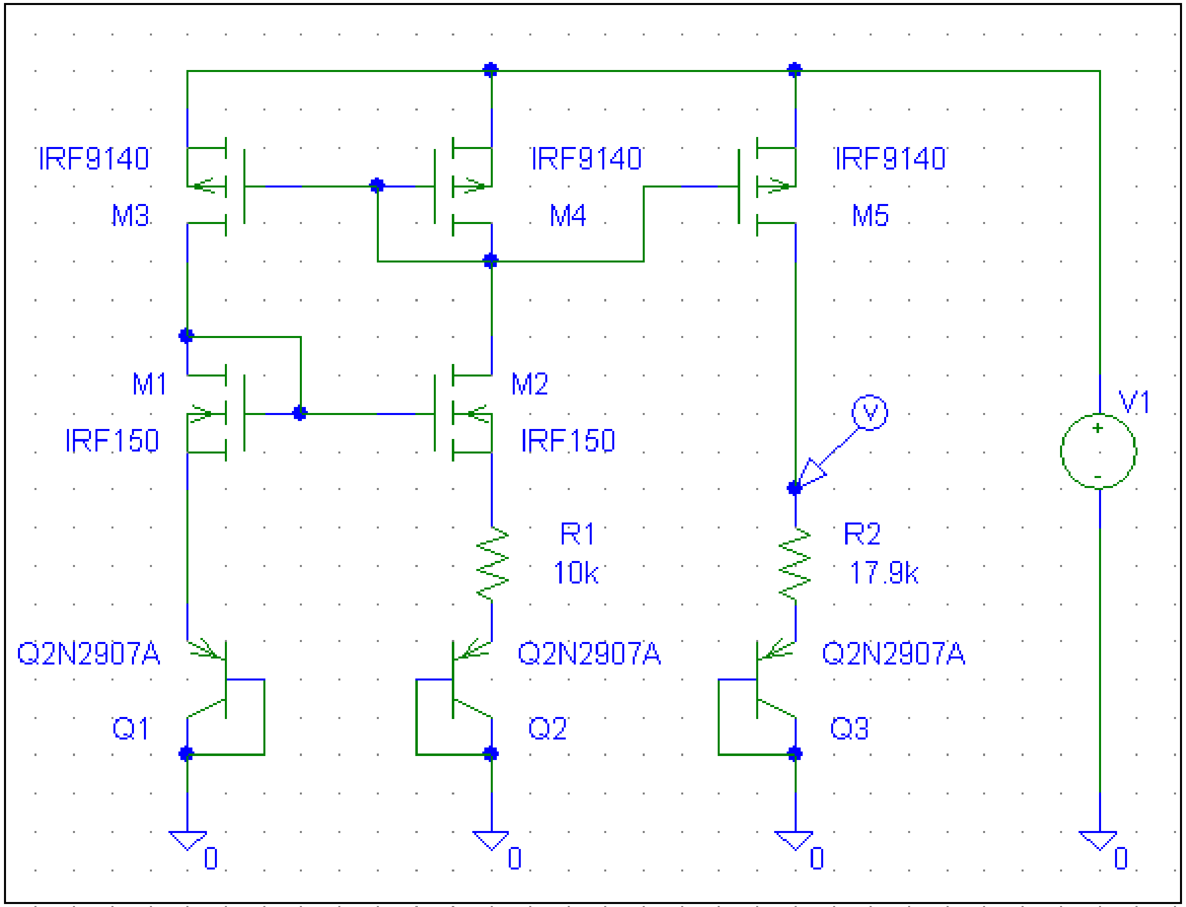
$$V_O(T) = |V_{BE_3}(T)| + \frac{R_2}{R_1}\frac{kT}{q}\ln\frac{I_{D6}}{I_{D7}}$$

$$\left.\begin{aligned} V_O(T) &= |V_{BE_3}(T)| + \frac{R_2}{R_1}\frac{kT}{q}\ln\left[\frac{(W/L)_6}{(W/L)_7}\right] \\ |V_{BE}(T)| &= A + BT + CT\ln\left(\frac{T}{T_0}\right) \\ B + \frac{R_2}{R_1}\frac{k}{q}\ln\left[\frac{(W/L)_6}{(W/L)_7}\right] &= 0 \end{aligned}\right\} \Rightarrow V_O(T) = A + CT\ln\left(\frac{T}{T_0}\right)$$

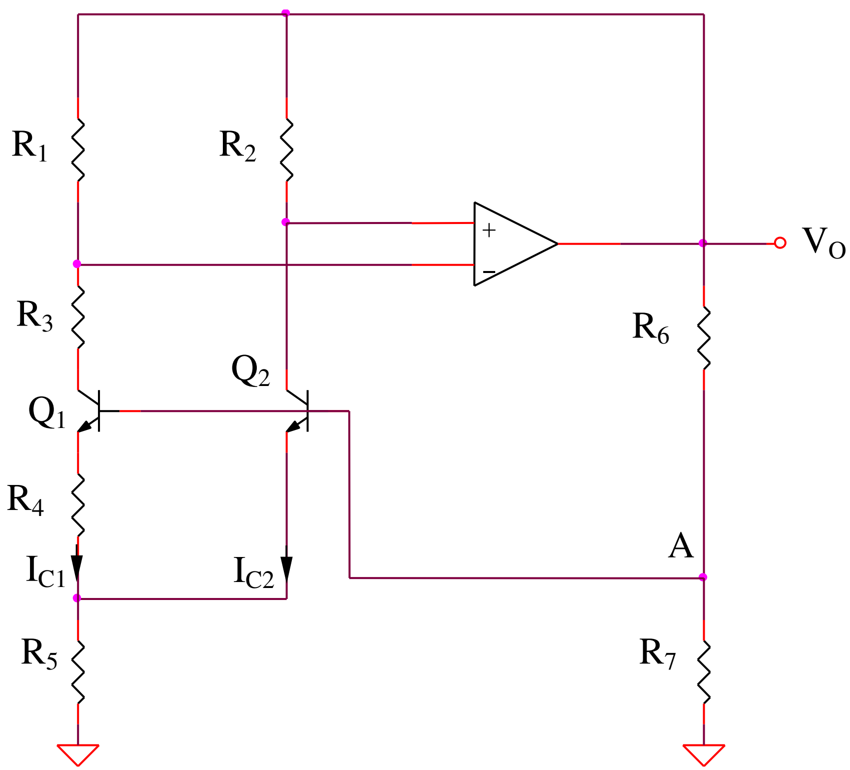
SIMULATION for CMOS voltage reference
Temperature dependence of the reference voltage

SIMULATION for CMOS voltage reference
Temperature dependence of the reference voltage

SIM 2.13: V_{D5} (t)



Exemple (3)



$$\left.\begin{aligned} I_{C1} &= \frac{V_{BE2}-V_{BE1}}{R_4} = \frac{V_{th}}{R_4}\ln\frac{I_{C2}}{I_{C1}} \\ I_{C1}R_1 &= I_{C2}R_2 \end{aligned}\right\} \Rightarrow$$

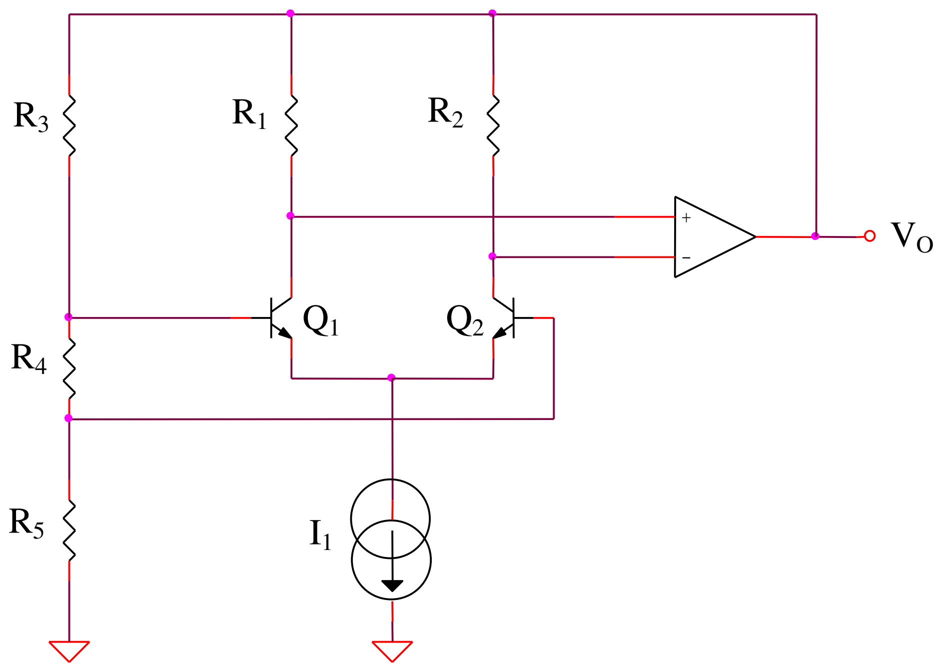
$$\Rightarrow I_{C1} = \frac{V_{th}}{R_4}\ln\frac{R_1}{R_2}$$

$$\left.\begin{aligned} V_A(T) &= (I_{C1}+I_{C2})R_5 + V_{BE2}(T) \\ V_A(T) &= V_O(T)\frac{R_7}{R_6+R_7} \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} &\Rightarrow V_O(T) = \left(1+\frac{R_6}{R_7}\right)\left[V_{BE_2}(T) + \frac{R_5}{R_4}\left(1+\frac{R_1}{R_2}\right)V_{th}\ln\left(\frac{R_1}{R_2}\right)\right] \\ &\frac{R_5}{R_4}\left(1+\frac{R_1}{R_2}\right)\frac{k}{q}\ln\left(\frac{R_1}{R_2}\right) + B = 0 \end{aligned}\right\} \Rightarrow V_O(T) = \left(1+\frac{R_6}{R_7}\right)\left[A + CT\ln\left(\frac{T}{T_0}\right)\right]$$

Derived circuits: temperature sensors

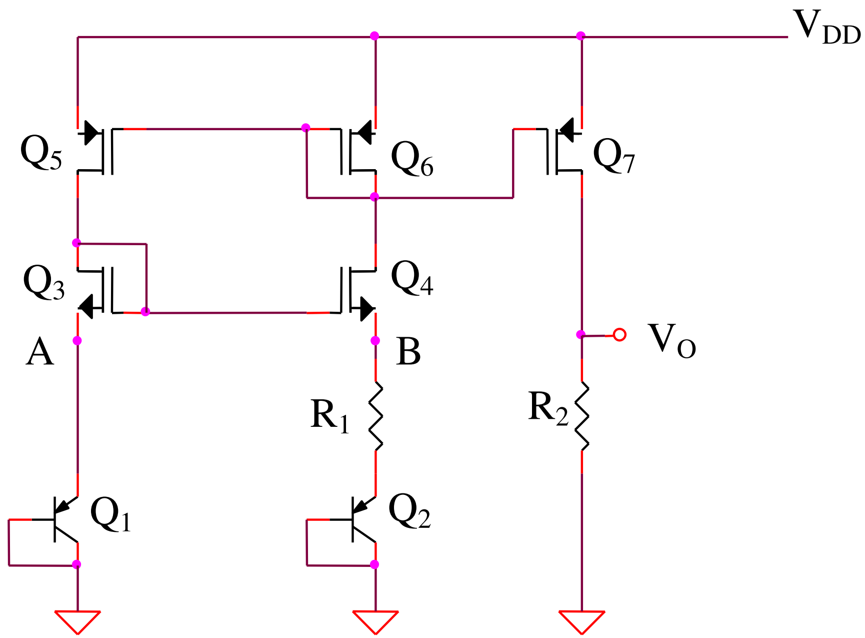
Example (1)



$$V_O(T)\frac{R_4}{R_3+R_4+R_5}=V_{BE1}-V_{BE2}=V_{th}\ln\frac{I_{C1}}{I_{C2}}=V_{th}\ln\frac{R_2}{R_1}\Rightarrow$$

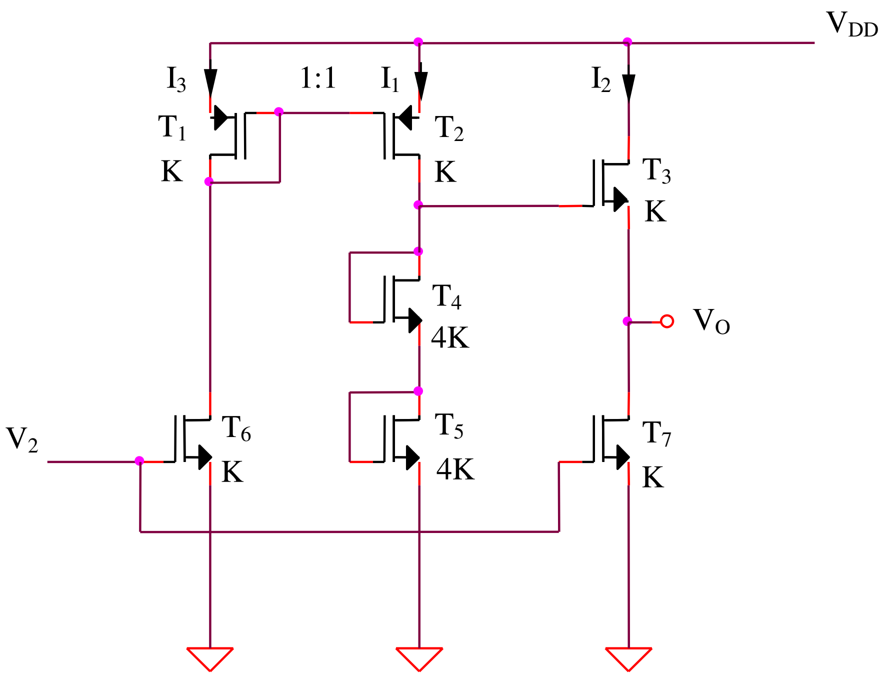
$$\Rightarrow V_O(T)=\left(1+\frac{R_3+R_5}{R_4}\right)V_{th}\ln\left(\frac{R_2}{R_1}\right)=ct.T$$

Example (2)



$$V_O = R_2 I_{D7}(T) = R_2 I_{D4}(T) = R_2 \frac{|V_{BE_1}| - |V_{BE_2}|}{R_1} = \frac{R_2}{R_1} V_{th} \ln\left[\frac{(W/L)_5}{(W/L)_6}\right] = ct.T$$

Example (3) – the threshold voltage extractor circuit



$$V_O = 2V_{GS_4} - V_{GS_3} = 2\left(V_T + \sqrt{\frac{2I}{4K}}\right) - \left(V_T + \sqrt{\frac{2I}{K}}\right) = V_T = V_{T0} + a\left(T - T_0\right)$$